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(54) System and method of robust sequence estimation in the presence of channel mismatch conditions.

(57) An adaptive communications receiver for data-transmission which comprises a channel estimator and sequence estimator. The channel estimator consists of a recursive least squares algorithm which uses a known training signal or past decisions to provide a channel estimate. The sequence estimator consists of a new recursive algorithm which uses the channel estimate to estimate the information sequence transmitted over the channel. The receiver exploits the commonality between the sequence and channel estimator to provide an efficient implementation. The receiver can have varying degrees of non-linearity and is lower in complexity and more robust than linear or decision feedback equalizers in the presence of channel mismatch.

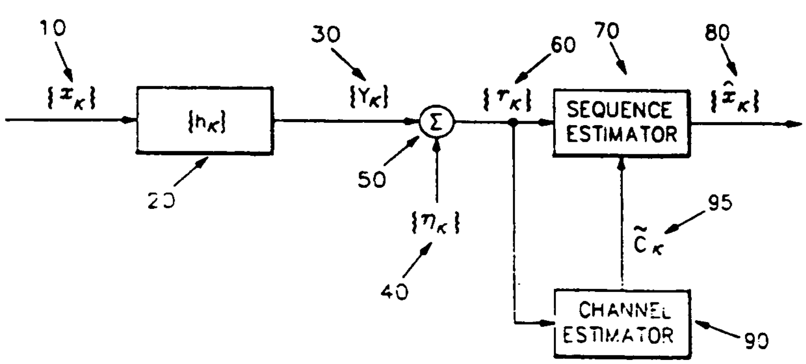

FIG.1

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FIELD OF THE INVENTION

The present invention relates to receivers which are used in voice and data communications in applications such as multi-media, modem, optical and magnetic storage devices, and wireless systems. In particular, it relates to a system and method of robust sequence estimation in the presence of channel mismatch conditions.

CROSS REFERENCE TO RELATED APPLICATIONS

The present application claims priority from and is related to the following co-pending Patent Application:
US Patent Application Serial No.07/846,651, filed March 5, 1992, entitled "System and Method of Estimating Equalizer Performance in the Presence of Channel Mismatch," of F. Gozzo.

This co-pending application and the present application are owned by one and the same assignee, namely International Business Machines Corporation of Armonk, New York. The description set forth in the co-pending application is incorporated into the present application by this reference.

SHORT GLOSSARY OF TERMS

Term	Definition
AWGN	Additive White Gaussian Noise
BER	Bit Error Rate

BLSCE	Batch Least Squares Channel Estimator
BLSSE	Batch Least Squares Sequence Estimator
Channel	A channel is any physical medium or media for connecting a transmitter and receiver. Functions which are included in transmitter and/or receiver can also be considered part of the channel. Physical media can be air, wire, cable, optical fiber, a magnetic disk, etc.
DFE(L,M)	A decision feedback equalizer which has L forward taps and M feedback taps, all of which are assumed to be optimized for an MSE criterion.
Equalizer	A form of an ISI receiver for channel communication.
KFE	Kalman Filter Equalizer
ISI	Intersymbol Interference
ISI Receiver	A receiver which attempts to eliminate intersymbol interference.
LFE(L,M)	A linear equalizer which has L forward taps and M feedback taps, all of which are assumed to be optimized for an MSE criterion.
Mismatch	A receiver which has been optimized for one channel, yet operates under a different channel, is said to operate in the presence of channel mismatch.
MSE	Mean square error
PAM	Pulse Amplitude Modulation
RLS	Recursive Least Squares
RLSCE	Recursive Least Squares Channel Estimation
RLSSE	Recursive Least Squares Sequence Estimation

RLSSE(L,D) Recursive Least Squares Sequence Estimator with decision depth L and D bits constrained per iteration. It is illustrated by FIGURE 6.

SNR Signal to Noise Ratio

REFERENCES USED IN THE DISCUSSION OF THE INVENTION

In the detailed description which follows, the following works will be referenced as an aid for the reader.

US PATENT DOCUMENTS

U.S. Patent 4,985,902, issued January 2, 1991, of M.K. Gurcan, entitled "Decision Feedback Equalizer and Method of Operating A Decision Feedback Equalizer,"
U.S. Patent 4,539,690, issued September 3, 1985, of J. Speidel entitled "Automatic and Adaptive Digital Nonlinear Decision Feedback Equalizer," and
U.S. Patent 4,577,329, issued March 18, 1986, of R. Brie et al., entitled "Self-Adaptive Equalizer For Baseband Data Signals", all
are of incidental interest because they relate to a method which attempts to provide sequence estimates based on a Decision Feedback Equalizer. My investigation has shown that the assumptions which underlie those reports are not always valid and that the performance of the present invention can surpass that of the aforementioned patents in many practical situations.

OTHER PATENT DOCUMENTS

UK Pat App 8523286, by Mulgrew and Cowan, September, 1985, entitled "An adaptive infinite impulse response equaliser," is another item of incidental interest and is discussed in the background section.

OTHER PUBLICATIONS

[1] R.E. Lawrence and H. Kaufman, "The Kalman Filter for the Equalization of a Digital Communication Channel," *IEEE Trans. Commun. Tech.,* Vol. COM-19, No. 6, pp 1137-1141, Dec. 1971.
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[4] A. Luvison and G. Pirani, "A State-Variable Approach to the Equalization of Multilevel Digital Signals," *CSELT Rapporti Tecnici,* No. 3. pp. 3-22, April 1974.
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[8] S. Prasad and S.S. Pathak, "An Approach to Recursive Equalization via White Sequence Estimation Techniques," *IEEE Trans. Commun.,* Vol. COM-35, No. 10, pp. 1037-1040, Oct. 1987.
[9] M.K. Yurtseven and M.M. Kumar, "An Adaptive FIR Kalman Equalizer With Improved Convergence Properties," in *Proc. of Annual Pitts. Conf. on Computer Sim.,* Vol. 21, pt. 2, pp. 507-513, 1990.
[10] E. Delle Mese and G. Corsini, "Adaptive Kalman Filter Equalizer," *Elec. Letters,* Vol. 16, No. 14, pp.547-549, July 3 1980.

BACKGROUND OF THE INVENTION

It would be of great use to provide a receiver which can be employed in many applications which include, but are not limited to, mobile cellular communications, satellite communications, indoor wireless networks, and multi-media applications (e.g. mass storage devices). Since the physical channel in these systems is susceptible to many variations, the receiver is usually operating with inaccurate channel estimates; thus, the robustness of the receiver is very important in these applications. The kind of receiver which I have created provides a robust sequence estimation capability with low complexity and attractive implementation characteristics. It is called an RLSSE receiver. Before detailing my detailed description, in which I will show that my RLSSE receivers can be implemented in an extremely efficient manner, since the same basic algorithm can be used for both the channel and sequence estimation tasks, I will summarize some past developments.

The Kalman Filter Equalizer (KFE)

The basic RLSSE structure is related to the Kalman filter equalizer (KFE) originally proposed by Lawrence and Kaufman [1], who utilized a discrete Kalman filter in order to estimate a binary input sequence transmitted over a dispersive FIR channel. They also treated the channel estimation problem by extending the state vector to include the channel tap coefficients. This (non-linear) extended Kalman-filter (EKF) approach falls into the category of blind equalization since no training sequence is exploited.

There have been several other efforts which follow the KFE approach for sequence estimation. Mark [2] briefly discusses the initialization of the state vector as well as a method of avoiding the EKF approach in [1] by estimating the channel tap gains via a pseudo-decision feedback approach. Kleibanov et al. [3] also investigated the same Kalman equalizer but further investigated its convergence properties and showed the advantages of increasing the filter dimension beyond the length of the channel filter. Luvison and Pirani [4] [5] also proposed a scheme which is based on that of Lawrence and Kaufman [1] but additionally discussed topics such as carrier recovery, timing extraction, and limitations of the KFE under real-time constraints.

An in-depth summary of the KFE as well as its practical considerations can be found in the work of Benedetto and Biglieri [6]. They extend the theory to cover correlated data and also provide several examples to demonstrate the sensitivity of the KFE to channel mismatch. Mulgrew and Cowan also present a detailed state-space analysis of the KFE for signalling over minimum and non-minimum phase channels [7] (see also UK Pat App 8523286, Sept 1985). By exploiting Wiener filter theory, [7] bounds the performance of the Kalman Filter Equalizer (KFE) and shows that the convergence and MSE performance of finite impulse response transversal filters and KFEs are roughly the same. They also found that the KFE was typically of lower order, although this does not necessarily constitute lower overall complexity.

Prasad and Pathak [8] develop a similar analysis in that a state-space approach is exploited. However, they use a smoothing approach from seismology and claim that it yields marginally better performance than the Kalman filter [1] under a low-complexity constraint. More recently, Yurtseven and Kumar [9] revisited the work of Kalman [1] and applied a stochastic Newton algorithm for the estimation of the extended state-vector. Although they claim to have improved on the EKF approach [1], their approach is also not guaranteed to converge. Similar efforts for improving the EKF channel estimator were also found in the work of Delle Mese and Corsini [10].

Limitations of the KFE Receiver

Although the KFE schemes described above are theoretically sound and may be useful in certain applications, they share two fundamental limitations. First, one must have an accurate estimate of the noise statistics represented as n_k 40 in FIGURE 1 in order to fully exploit the Kalman filter structure. This not only increases system complexity (since a noise variance estimator must be implemented), but it also renders KFE receivers susceptible to noise mismatch. Second, and more importantly, is the fact that the Kalman filter is inherently a *linear* filter, hence its performance over channels with spectral nulls will be severely degraded by noise-enhancement. In fact, the performance of all KFE receivers is (asymptotically) no better than conventional linear equalizers optimized for an MSE criterion. This fundamental limitation, coupled with the noise mismatch problem, precludes the use of existing KFE schemes over many practical channels.

Motivation Behind The RLSSE Receiver

The RLSSE receiver proposed herein has been motivated by three practical needs: Low-complexity, reduced sensitivity to channel mismatch, and adequate performance over channels with spectral nulls. My receiver differs from the KFE schemes in several fundamental respects. First, the error criterion used in the RLSSE scheme is based on a least squares criterion rather than a least mean square criterion. Since the least squares approach makes no assumptions regarding the statistics of the noise 40, I found that the RLSSE algorithms are impervious to white gaussian noise mismatch and that they are significantly less complex because an estimate of the noise variance is not needed.

Another fundamental difference is that one class of the proposed RLSSE algorithms is *nonlinear.* Therefore, while my simplest (linear) algorithm is comparable in performance to linear equalizers, and hence KFE receivers, my *constrained* RLSSE algorithm can cope with channels with spectral nulls. Finally, and perhaps most importantly from a practical standpoint, I will show that my algorithms lead to an extremely efficient implementation since the channel estimation and sequence estimation tasks have a duality that is fully exploited by the RLSSE approach.

SUMMARY OF THE INVENTION

In order to provide the quality needed for evaluation of a received signal and reduce the cost of the evaluation, I provide a common way to evaluate two kinds of elements affecting the received signal. The information sequence needs evaluation and the channel filter effect needs evaluation. In accordance with my invention I provide a receiver which evaluates a received signal. At an input port for a received signal which has the characteristics that need evaluation (including two signal sequences, one of which is an unknown information sequence and another of which is an unknown channel filter effect), the signal is carried to my receiver system. The receiver system provides an output estimate of the unknown information sequence, and a common functional evaluation unit for evaluating the received signal performing the functions of a sequence estimator and a means for providing a channel estimate. By using common functional steps a common functional unit can be provided, and embodied in a single chip.

In this invention I apply the theory of recursive least squares (RLS) algorithms to the problem of ***sequence estimation*** over intersymbol interference (ISI) channels. Specifically, I will propose a new family of low-complexity ISI receivers which are based on variants of the common RLS algorithm. These new algorithms, which I will refer to collectively as ***recursive least squares sequence estimation*** (RLSSE) algorithms, offer the performance and low complexity enjoyed by linear and DFE receivers, yet are insensitive to practical phenomena such as noise mismatch which was shown to plague both linear and DFE receivers in one patent application Serial No. 07/846,651 of March 5, 1992.

In accordance with the present invention a system and method is provided which provides a robust sequence estimation capability with low complexity and attractive implementation characteristics.

The receiver of the present invention is comprised of a channel estimator, which estimates the coefficients of the discrete-time equivalent channel filter (also known as a channel impulse response), and a sequence estimator which provides an estimate of the information sequence which was transmitted over the discrete-time equivalent channel.

The algorithms used by the channel estimator and sequence estimator of the present invention have been successfully implemented on a variety of computers including simple desktop and large mainframe systems. Based on the performance on these systems, implementing the present invention in real-time applications via common general-purpose processors is highly desirable.

My invention is important in many applications including the fields of mobile cellular communications, satellite communications, indoor wireless networks, and multi-media applications (e.g. mass storage devices).

BRIEF DESCRIPTION OF THE DRAWINGS

My detailed description explains the preferred embodiments of my invention, together with advantages and features, by way of example with reference to the following drawings.

FIGURE 1 shows schematically an overview of the popular discrete-time equivalent communication system model assumed by the present invention, and shows the receiver system which is comprised of a channel estimator and a sequence estimator.

FIGURE 2 shows the channel estimator 90 of FIGURE 1 in more detail, and specifically shows a particular channel estimator I will refer to as the batch least squares channel estimator (BLSCE) which

provides an estimate of the channel filter 20.

FIGURE 3 shows a more efficient version of the BLSCE device shown in FIGURE 2. This particular channel estimator is my recursive least squares channel estimator (RLSCE) which is the preferred embodiment for the channel estimator of the present invention.

FIGURE 4 shows the sequence estimator 70 of FIGURE 1 in more detail and specifically shows a particular sequence estimator I will refer to as the batch least squares sequence estimator (BLSSE).

FIGURE 5 shows a more efficient version of the linear BLSSE device shown in FIGURE 4. This recursive sequence estimator will be referred to as the linear recursive least squares sequence estimator and denoted as RLSSE(L,0).

FIGURE 6 shows the preferred embodiment for the sequence estimator of the present invention. This sequence estimator includes the linear receiver of FIGURE 5 but also extends it to the nonlinear case. I will refer to this receiver as RLSSE(L,D).

FIGURE 7 shows an expanded functional view of the D-threshold device 600 shown in FIGURE 6.

FIGURE 8 shows the magnitude spectrum of a particular discrete-time equivalent channel filter 20 as a function of the parameter *a*.

FIGURE 9 compares the performance of the present invention against linear and DFE receivers over the test channel shown in FIGURE 3 when there is no channel mismatch.

FIGURE 10 compares the performance of the present invention against linear and DFE receivers over the test channel shown in FIGURE 3 when there is white gaussian noise mismatch present.

FIGURE 11 compares the performance of the present invention against linear and DFE receivers over the test channel shown in FIGURE 3 when there is channel filter mismatch present.

FIGURE 12 compares the theoretical versus test performance of the present invention in the presence of white gaussian noise mismatch. The theoretical and experimental results for LFE and DFE receivers is also shown.

Tables 1-5 illustrate my invention. They are detailed at the end of the detailed description for convenient reference.

DETAILED DESCRIPTION OF THE INVENTION

Before considering my preferred embodiments in detail, it is worthwhile to briefly review the theoretical development and discuss a simple test example.

First, I provide a review of the channel estimation problem where I introduce notation and emphasize those points which are key to the RLSSE algorithms. The structure of the linear RLSSE algorithm is then developed, followed by an extension to the more generic nonlinear RLSSE receiver. Finally, I provide a theoretical analysis of the linear receiver, followed by a detailed empirical performance analysis of both linear and nonlinear versions. Implementation considerations are then briefly discussed, followed by a summary and concluding remarks.

Review of Least Squares Channel Estimation

Although the development of the least squares channel estimator 90 is now well known, I briefly review it here so that my RLSSE development can exploit this familiar derivation. I will derive both the batch and recursive forms of the least squares channel estimator 90 in order to expose the features which will help one to predict the performance of the RLSSE receiver. In addition, in order to deal with possible variations in the coefficients of the channel filter 20, an exponential weighting will also be employed in the least squares performance index.

Batch Least Squares Channel Estimator (BLSCE)

For generality, I assume the dynamics of the discrete-time equivalent channel filter 20 can be described by a finite-order difference equation given by

$$y_k = \sum_{i=1}^{M} a_i y_{k-i} + \sum_{i=0}^{L-1} b_i x_{k-i} \qquad (1)$$

where x_k and y_k represent the noise-free channel filter input 10 and output 20, respectively, and M and L represent the order of the model. Thus, it is clear that y_k 30 can be recursively computed based solely on M past outputs and L inputs, and my development also holds for the special FIR case (i.e., $M = 0$).

I can define the coefficients of the actual channel filter in (1) by **a** and **b**, which are of dimension $M \times 1$ and $L \times 1$, respectively, and are given by

$$\mathbf{a} = [a_1\ a_2\ \dots\ a_M]^T$$
$$\mathbf{b} = [b_0\ b_1\ \dots\ b_{L-1}]^T \quad (2)$$

where $[\ \bullet\ ]^T$ denotes transposition. For notational simplicity, an $(M + L) \times 1$ *coefficient vector* is introduced and defined as

$$\mathbf{c} = \begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix} \quad (3)$$

Since the vector **c** completely specifies the actual channel filter 20, the channel estimation function can be posed as finding an estimate, $\tilde{\mathbf{c}}$, which is closest to **c** in a least squares sense.[1] The actual channel filter 20 and its estimate at time k will be denoted by $\mathbf{c}_k$ and $\tilde{\mathbf{c}}_k$ 95, respectively.

Assume that a ***known*** training sequence of $N + 1$ symbols is transmitted, resulting in $N + 1$ input-output observable pairs,

$$\{(x_0, r_0)\ (x_1, r_1)\ \dots\ (x_N, r_N)\} \quad (4)$$

[1] Real-valued (soft) estimates of a vector $\mathbf{z}$ will be denoted as $\tilde{\mathbf{z}}$. In the sequence estimator 70 development, $\hat{\mathbf{z}}$ will denote a binary (hard) estimate. Also, unless denoted otherwise by context, lowercase bold symbols represent vectors, while uppercase bold symbols represent matrices.

In order to ensure a unique solution, let $N > n$ where n is defined as $n = \max(L,M)$. Let the vector $\mathbf{q}_k$ be defined as the $(M + L) \times 1$ *training vector* 330[2]

$$\mathbf{q}_k = [\ y_{k-1}\ y_{k-2}\ \dots\ y_{k-M}\ x_k\ x_{k-1}\ \dots\ x_{k-(L-1)}]^T \quad (5)$$

I define the estimation error, e_k, as the difference between the actual channel output (i.e., the desired response) and the estimated channel output,

$$e_k = r_k - \mathbf{q}_k^T \tilde{\mathbf{c}}_k \quad (6)$$

I can use the vector form of the above equation, i.e.

$$\mathbf{e}_N = \mathbf{r}_N - \mathbf{Q}_N \tilde{\mathbf{c}}_N \quad (7)$$

where $\mathbf{r}_N$ 230 and $\mathbf{e}_N$ are $(N - n + 1) \times 1$ *observable* and *error* vectors, respectively, and $\mathbf{Q}_N$ is an $(N - n + 1) \times (M + L)$ *training matrix*. These are given by

$$\mathbf{r}_N = [r_n \; r_{n+1} \; \cdots \; r_N]^T$$
$$\mathbf{e}_N = [e_n \; e_{n+1} \; \cdots \; e_N]^T$$
$$\mathbf{Q}_N = \begin{bmatrix} \mathbf{q}_n^T \\ \mathbf{q}_{n+1}^T \\ \vdots \\ \mathbf{q}_N^T \end{bmatrix} \qquad (8)$$

[2] For IIR channels ($M > 0$), the first M elements in $\mathbf{q}_k$ are not observable, even with known training signals. An estimate of $\mathbf{q}_k$, such as $\mathbf{q}_k = [\, r_{k-1} \; r_{k-2} \; \cdots \; r_{k-M} \; x_k \; x_{k-1} \; \cdots \; x_{k-(L-1)}]^T$ must be used during implementation. Although the implications of this are not significant at moderate SNR, this will, in general, lead to biased channel estimates for IIR adaptive filters based on an output-error formulation. Thus, my preferred embodiment utilizes an FIR channel filter model.

The cost, J, for a weighted least squares performance index can be expressed as

$$J = \sum_{k=n}^{N} w_k \, e_k^2 \qquad (9)$$

where w_k will be defined as an exponential weighting term,

$w_k = \gamma^{(N-k)}$; $0 < \gamma \leq 1$, $n \leq k \leq N$ (10)

In order to express my cost equation in matrix form, I can define a weighting matrix, $\mathbf{W}_N$, whose elements are given by

$$\mathbf{W}_N(i,j) = \begin{cases} w_{n+i-1} & ; \; i=j \\ 0 & ; \; i \neq j \end{cases} \qquad (11)$$

Thus, my cost function can be formulated as:

$$\begin{aligned} J &= \mathbf{e}_N^T \mathbf{W}_N \mathbf{e}_N \\ &= [\mathbf{r}_N - \mathbf{Q}_N \tilde{\mathbf{c}}_N]^T \mathbf{W}_N [\mathbf{r}_N - \mathbf{Q}_N \tilde{\mathbf{c}}_N] \\ &= \mathbf{r}_N^T \mathbf{W}_N \mathbf{r}_N - 2\, \tilde{\mathbf{c}}_N^T \mathbf{Q}_N^T \mathbf{W}_N \mathbf{r}_N + \tilde{\mathbf{c}}_N^T \mathbf{Q}_N^T \mathbf{W}_N \mathbf{Q}_N \tilde{\mathbf{c}}_N \end{aligned} \qquad (12)$$

Now, by using the gradient operator to minimize (12) with respect to $\tilde{\mathbf{c}}_N$, I get

$$\nabla J = -2\mathbf{Q}_N^T \mathbf{W}_N \mathbf{r}_N + 2\, \mathbf{Q}_N^T \mathbf{W}_N \mathbf{Q}_N \tilde{\mathbf{c}}_N = 0 \qquad (13)$$

where **0** is the all-zero vector. Solving for $\tilde{c}_N$ yields the optimal weighted least squares channel estimate.

Optimal Weighted Least Squares Estimator

$$\tilde{\mathbf{c}}_N^{LS} = \left[\mathbf{Q}_N^T \mathbf{W}_N \mathbf{Q}_N\right]^{-1} \mathbf{Q}_N^T \mathbf{W}_N \mathbf{r}_N \qquad (14)$$

By inspecting (14), one sees that by continuously updating the observable vector **r** 230 and the training matrix **Q**, I can update my channel estimate accordingly. This can be accomplished by updating $\mathbf{r}_k$ 230 as follows

$$\mathbf{r}_k^T = \mathbf{r}_{k-1}^T \mathbf{S} + r_k \mathbf{g} \qquad (15)$$

where I have introduced an initialization vector, **g** 200, which is defined by

g = [0 0 ... 0 0 1] (16)

and the *shift matrix*, **S** 210, given by

$$\mathbf{S} = \begin{bmatrix} 0 & 0 & & 0 & 0 \\ 1 & 0 & & 0 & 0 \\ 0 & 1 & \cdots & \vdots & \vdots \\ \vdots & \vdots & & 0 & 0 \\ 0 & 0 & & 1 & 0 \end{bmatrix} \qquad (17)$$

A block diagram[3] of the BLSCE structure is shown in FIGURE 2. Note that while the continuous update of the observable vector, **r** 230, is explicitly shown, the continuous update of the training matrix, **Q**, is not shown. Several observations can now be made regarding the BLSCE structure which will be useful for comparing with the batch sequence estimator 70 development:

BLSCE Observations

1. **Linear Estimator** - Since all operations on the observable vector 230 are linear operations, I see that the BLSCE structure is clearly a linear estimator.
2. **Time-Varying** - Since

$$\mathbf{q}_k^T,$$

and hence

$$\mathbf{Q}_k^T,$$

[3] Solid lines in diagrams represent scalar values, while dashed lines represent vector quantities.

are changing with each new training symbol, the filter shown in the diagram is a time-varying structure. Note further that this is true whether the channel is time-variant or not.

3. **High Complexity** - Since the BLSCE filter is guaranteed to be time-variant, the inverse operation shown in the diagram must be performed with each new sample. This is, of course, highly inefficient and so recursive forms are desirable.

Recursive Least Squares Channel Estimator (RLSCE)

The recursive implementation of (14) is embodied in FIGURE 3. The figure illustrates a linear recursive estimator which forces the estimate to take the form

$$\tilde{\mathbf{c}}_k = \tilde{\mathbf{c}}_{k-1} + \mathbf{k}_k \eta_k \qquad (18)$$

where η_k represents the estimation error, and $\mathbf{k}_k$ 310 is a gain vector. Thus,the heart of the recursive solution lies in finding the gain vector which yields the least squares estimate.

A summary of the key equations in the RLSCE algorithm are given in TABLE 1. Note that the algorithm requires an initialization of $\tilde{c}_k$ 95 by an initial guess, as well as a starting point for the correlation matrix inverse

$$\mathbf{P}_k^{-1} = \mathbf{Q}_k^T \mathbf{W}_k \mathbf{Q}_k$$

to remedy its possibly ill-conditioned nature. Also, the intermediate variables, $\mathbf{U}_k$ and $\mathbf{v}_k$, have been introduced to reduce computations.

Least Squares Sequence Estimation

Before I develop the RLSSE algorithm, it is worthwhile to step back and contrast the channel and sequence estimation problems. The motivation behind the RLSSE receiver is more apparent once one sees the strong duality which exists between these two problems.

Duality Between Channel and Sequence Estimation

Theoretically, the receiver has no knowledge of whether a signal $\{x_k\}$ was passed through a linear filter with impulse response $\{h_k\}$, or, whether $\{h_k\}$ was passed through a filler with impulse response $\{x_k\}$. The channel output is simply the noise-corrupted convolution of the channel input 10 and channel filter 20,

$r_k = x_k * h_k + n_k$ (19)

By posing both problems in a system identification framework, I can describe the two problems as *dual* adaptive filtering tasks. That is, the channel estimation process uses a known information sequence to identify an unknown channel while the sequence estimation process uses a "known" channel filter 20 to identify an unknown sequence.

The inherent similarity between channel and sequence estimation is clearly evident from (19). Several observations worth noting are:

1. Both problems are inverse problems (more specifically, deconvolution problems) which are known to be *ill−posed.*
2. In general, accurate knowledge of $\{h_k\}$ 20 is required to estimate $\{x_k\}$ 10, and vice-versa.
3. Accurate estimation of an arbitrary $\{h_k\}$ 20 requires the use of a known sequence $\{x_k\}$ 10 which is *persistently exciting,* and vice-versa.

Although both problems share these similarities, the fact remains that $\{h_k\}$ 20 is specified over a field of real numbers, and $\{x_k\}$ 10 is over a binary field. Thus, while channel estimation deals with the estimation of a *finite* number of *real valued* parameters, sequence estimation deals with the estimation of an *infinite* number of *binary valued* parameters.

Batch Least Squares Sequence Estimator (BLSSE)

The duality described above suggests that by making several modifications, the same estimator structure can be used. I now revisit the batch channel estimator 90 and perform the appropriate modifications.

First, an appropriate model for the unknown information sequence must be found. Since it is assumed that $\{x_k\}$ 10 is an uncoded sequence[4], the z-transform of an arbitrary L-length message sequence can be given by a simple FIR model

$$X(z) = \sum_{i=0}^{L-1} x_i z^{-i} \qquad (20)$$

Thus, by using the same naming convention followed in the channel estimation development, I find that the coefficient vector and its sort estimate are now given by

$$\begin{aligned} \mathbf{c}_k &= [\, x_{k-(L-1)} \; x_{k-(L-1)+1} \; \cdots \; x_{k-1} \; x_k \,]^T \\ \tilde{\mathbf{c}}_k &= [\, \tilde{x}_{k-(L-1)} \; \tilde{x}_{k-(L-1)+1} \; \cdots \; \tilde{x}_{k-1} \; \tilde{x}_k \,]^T \end{aligned} \qquad (21)$$

Next, I must reexamine the topic of input-output observable pairs. Recall that the channel estimator 90 development required the use of a known training sequence. The same applies for sequence estimation. However, now the "known" training signal is the estimate of the channel filter 20. Thus, the training vector 330 is now given by the (Lx1) vector

$$\mathbf{q}_k = [\, \tilde{h}_{L-1} \; \tilde{h}_{L-2} \; \cdots \; \tilde{h}_1 \; \tilde{h}_0]^T \qquad (22)$$

where the estimated samples of the channel filter 20 are determined by

$$\tilde{h}_k = \sum_{i=1}^{M} \tilde{a}_i h_{k-i} + \sum_{i=0}^{L-1} \tilde{b}_i \delta_{k-i} \qquad (23)$$

where δ_{k-i} is the Dirac delta function, and the parameters $\tilde{a}_i$ and $\tilde{b}_i$ were found by the channel estimator 90. Note that the training matrix also changes with the new $\mathbf{q}_k$ as defined in (8).

A summary of these key variables and their counterparts in the channel estimation process are shown in TABLE 2.

By exploiting the duality shown above and substituting directly in (14)

$$\tilde{\mathbf{c}}_k = [\mathbf{Q}_k^T \mathbf{W}_k \mathbf{Q}_k]^{-1} \mathbf{Q}_k^T \mathbf{W}_k \mathbf{r}_k \qquad (14)$$

I obtain the *unconstrained* least squares solution, i.e., the *real-valued* parameter vector which is closest to the actual parameter vector in a least squares sense. Recall, however, that I am interested in an infinite-

[4] This model could be easily modified for sequences which are coded by a known linear filter.

length binary-valued sequence. In order to accomplish this, I first extract the first element of $\tilde{\mathbf{c}}_k$ 95 by filtering the parameter vector to obtain

$$\tilde{x}_{k-(L-1)} = \mathbf{f}\,\tilde{\mathbf{c}}_k \tag{24}$$

where the *pick-off vector*, **f** 400, is defined by

$\mathbf{f} = [\,1\ 0 \ldots 0]$ (25)

This soft-decision can now be passed through a simple threshold device 420 to yield the hard-decision, $\hat{x}_{k-(L-1)}$ 80, i.e.

$$\hat{x}_{k-(L-1)} = \operatorname{sgn}\left\{\tilde{x}_{k-(L-1)}\right\} = \begin{cases} +1 & ; \ \tilde{x}_{k-(L-1)} \geq 0 \\ -1 & ; \ \tilde{x}_{k-(L-1)} < 0 \end{cases} \tag{26}$$

The process described would continue by simply shifting the elements of $\mathbf{r}_k$ as was done in the BLSCE structure and recomputing the least squares solution g iven by (14). A block diagram of the BLSSE receiver is shown in FIGURE 4.

***BLSSE** Observations*

1. **Identical in Structure To Batch Channel Estimator** - Up to the pick-off vector, **f** 400, shown in FIGURE 4, the BLSSE and BLSCE struct ures are identical in form; hence, by simply redefining the contents of the training vector 330, one can use the same architecture to estimate both the channel and unknown sequence.
2. **Linear Estimator** - As was the case for the BLSCE structure, the *soft* estimate is a linear function of the observable vector 230; hence, the BLSSE estimator, up to the threshold device 420, is a linear receiver.
3. **Time-Invariant Receiver For Time-Invariant Channels** - Recall that the batch channel estimator 90 was time-varying, even for stationary channel filters. If the channel filter varies with time, then the BLSSE receiver will also be time-varying. However, if the channel filter is time-invariant, the training vector

$$\mathbf{q}_k^T$$

330, and hence

$$\mathbf{Q}_k^T,$$

of the BLSSE receiver will not change with each new training symbol. Therefore, once an initial burst of symbols has been sent, the sequence estimator reaches steady-state and hence becomes stationary.
4. **Moderate Complexity** - Since the sequence estimator 70 is time-invariant for time-invariant channel filters 20, the matrix

$$[\mathbf{Q}_k^T\mathbf{W}_k\mathbf{Q}_k]^{-1}\mathbf{Q}_k^T\mathbf{W}_k$$

must be calculated just once; hence, the batch form itself is not as inefficient as the BLSCE structure. However, even though the BLSSE structure may be more efficient than its BLSCE counterpart under time-invariant conditions, the ability to handle time-varying channel filters 20 still necessitates a recursive form.

Recursive Least Squares Sequence Estimator (RLSSE)

The recursive form of the least squares sequence estimator 70 follows the exact derivation of the recursive channel estimator 90. The only difference is at the end of each iteration where the following must occur in order to accommodate the shirt and threshold 420 operations:

1. **Pick Off and Constrain Soft Estimate** - Prior to shifting the elements of the coefficient vector, I must first extract an estimate of the "oldest" symbol, i.e., $\tilde{x}_{k-(L-1)}$. Analytically, I can do so by filtering the parameter vector by the pick-off vector **f** 400 defined in (25). Once this sort estimate has been removed from the coefficient vector, I constrain it and announce the final estimate.

2. **Shift and Initialize** $\tilde{c}_k$ - By noting the form of the parameter vector, it is clear that after each iteration, a new initial estimate of the parameter vector can be found by simply shifting the elements to the left by using the shifting matrix, **S** 210, already defined in (17). In addition, while all RLS algorithms require an initial estimate of the coefficient vector at start-up, my RLSSE scheme requires a continuous initialization of the current (rightmost) symbol of the coefficient vector. This can be trivially accomplished by augmenting the shifted coefficient vector by the quantity $\tilde{x}_k\mathbf{g}$, where $\tilde{x}_k$ is an initial guess of the current symbol. Note that for my equally-likely binary model, $\tilde{x}_k = 0$, but I leave this important step in the algorithm for generality and for possible extensions to coded schemes.

Note that the shift matrix is a linear operation which is useful for the purpose of analysis, but *in practice this is a trivial memory shift* which simply propagates a particular vector element from right to left. Thus, for a filter of length five, the unconstrained parameter vector and its shifted/initialized version could be generally described by

$$\begin{aligned} \tilde{\mathbf{c}}_k^T &= [\ \tilde{c}_1\ \tilde{c}_2\ \tilde{c}_3\ \tilde{c}_4\ \tilde{c}_5\ ] \\ \tilde{\mathbf{c}}_k^T\mathbf{S} + \tilde{x}_k\mathbf{g} &= [\ \tilde{c}_2\ \tilde{c}_3\ \tilde{c}_4\ \tilde{c}_5\ \tilde{x}_k\ ] \end{aligned} \qquad (27)$$

3. **Shift and Initialize** $\mathbf{P}_k$ - The inverse covariance matrix must also be shifted (up and to the left) to remain synchronized with the coefficient vector. This can be accomplished by filtering the covariance matrix by the transformation $\mathbf{S}^T\mathbf{P}\mathbf{S}$ where **S** 210 is the previously defined shifting matrix. It should be noted that this covariance shift effectively shifts the elements of $\mathbf{k}_k$ 310 as well.

The "new" elements of $\mathbf{P}_k$ which correspond to the new symbol must also be initialized with each incoming symbol. I can satisfy this initialization by augmenting the shifted covariance matrix above with the matrix $\epsilon^{-1}\mathbf{gI}$ whose entries are all zero except for the lower-right element which has the value ϵ^{-1}. For example, shifting and initializing an arbitrary 3x3 matrix, **P**, would appear as

$$\begin{aligned} \mathbf{P} &= \begin{bmatrix} P(1,1) & P(2,1) & P(3,1) \\ P(1,2) & P(2,2) & P(3,2) \\ P(1,3) & P(2,3) & P(3,3) \end{bmatrix} \\ \mathbf{S}^T\mathbf{P}\mathbf{S} + \varepsilon^{-1}\mathbf{gI} &= \begin{bmatrix} P(2,2) & P(3,2) & 0 \\ P(2,3) & P(3,3) & 0 \\ 0 & 0 & \varepsilon^{-1} \end{bmatrix} \end{aligned} \qquad (28)$$

A block diagram of the linear RLSSE receiver is shown in FIGURE 5. For reasons which will be apparent in the next section, this receiver will be referred to as the RLSSE(L,0) receiver.

Constraining the Estimator - The RLSSE(L,D) Algorithm

The DFE is undoubtedly the most popular nonlinear receiver which exhibits both low complexity and reasonable performance over channels with spectral nulls. Furthermore, since the DFE is merely a nonlinear variant of the linear feedback equalizer, I have created a nonlinear variant of the linear RLSSE estimator in an effort to improve its performance over several practical channels with severe ISI.

Toward that end, consider the role of the past sequence estimate, $\tilde{c}_{k-1}$ 320, shown in FIGURE 5. During each iteration, this vector acts as an initial guess which will be corrected based on the error 300 and gain vector 310. Note, however, that $\tilde{c}_{k-1}$ 320 represents a ***soft*** estimate of the true coefficient vector $\mathbf{c}_k$; hence, I already know that it is, in general, not correct. Thus, it seems intuitive to constrain these sort estimates before they are used in the next iteration. If these decisions are correct, then $\tilde{c}_k$ 95 stands a good chance of being correct as well. However, since some of these elements have been in the RLSSE decoding window for only a few samples and are furthermore noise-corrupted, an incorrect decision certainly is possible. The leftmost element of $\tilde{c}_{k-1}$ 320, however, can be considered the most ***mature*** estimate, i.e., it has propagated through the full length of the RLSSE window and so it seems reasonable to assume that, after some point, it has been equalized to some extent such that constraining it may improve the overall performance.

In this spirit, I extend the linear RLSSE receiver to a new nonlinear structure which I call the RLSSE(L,D) receiver, as shown in FIGURE 6.

In the figure, all elements are identical to those found in FIGURE 5 except the new ***D−threshold*** device 600 which has been inserted in the feedback path. The *'D'* which is in the lower-right corner of the device signifies that only the leftmost D symbols will be constrained. Thus, the RLSSE(L,0) is the linear receiver already discussed, while, at the other extreme, the RLSSE(L,L) receiver uses a fully-constrained estimate. An expanded view of the D-threshold device 600 is shown in FIGURE 7 and a summary of the steps in the RLSSE(L,D) algorithm is given in TABLE 3.

Observations on RLSSE(L,D) Receiver

1. **Varying Degrees of Nonlinearity** - By noting once again that the final soft estimate is linear in the observable vector 230, note that for $D = 0$, the RLSSE is a linear estimator up to the threshold device 420. However, one can also increase D up to the length of the RLSSE window to increase the degree of nonlinearity.
2. **Same Core Algorithm As RLSCE** - It is important to note that the processing required by STEPS 0-8 of the RLSSE algorithm are identical to those found in the RLSCE algorithm, bat the definition of $\mathbf{q}_k$ has changed. Furthermore, STEPS 0-8 represent virtually all of the significant processing. This significant fact suggests that an RLSSE-based receiver can exploit the same core architecture for both the channel and sequence estimation tasks as I have successfully accomplished during extensive tests.
3. **Low to Moderate Complexity** - As shown in TABLE 3, the RLSSE algorithm can handle time-varying channels since $\mathbf{q}_k$ need only be updated with the new channel estimate. From the complexity analysis shown in TABLE 3, I see that the worst-case complexity is $O(N^2)$ for time-varying channels. If, however, the channel is time-invariant, the gain vector rapidly converges to its steady-state value. Therefore, the burdensome task of computing $\mathbf{P}_k$ is not necessary, and so STEPS 2-7 and STEP 12 can be avoided, resulting in an $O(N)$ algorithm.
4. **Synchronous Operation** - After the initial burst of L symbols have been received, the algorithm will synchronously generate estimates. Thus, the overhead of buffer management required by some receivers is avoided.

Theoretical Performance Analysis

An informative theoretical analysis of the nonlinear RLSSE(L,D) receiver does not seem feasible without unrealistic assumptions; hence, computer simulations have been used to establish the performance of the generic RLSSE(L,D) receiver. However, by assuming stationary and ergodic conditions, the theoretical performance of the linear RLSSE(L,0) structure can be easily determined, as is shown below.

Perfect Channel Knowledge

If one assumes that the channel estimator 90 generated a perfect estimate of the channel filter 20, the mean square error (MSE) performance of the linear RLSSE(L,0) structure can be readily ascertained. Since the least squares and least mean square error are asymptotically identical in AWGN, the asymptotic performance of the RLSSE(L,0) structure with perfect channel knowledge is that of all linear receivers which were optimized for an MSE criterion which is

MSE of RLSSE(∞,0) Receiver in White Gaussian Noise

$$J_{RLSSE(\infty,0)} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{N_0}{|H(e^{j\omega})|^2 + N_0} \, d\omega \tag{29}$$

where $|H(e^{j\omega})|^2$ represents the discrete power spectrum of the channel filter 20 and N_0 is the variance of the noise 40.

White Gaussian Noise Mismatch

If the variance of the noise 40 during training differs from that during decoding, a noise mismatch condition is present. However, recall that the RLSSE approach does not depend on the noise statistics, either implicitly or explicitly. It is strictly based on the channel estimate. Therefore, as long as the training sequence is sufficiently long such that the channel estimate converges to an unbiased solution, the performance of the RLSSE(*L,D*) algorithm during decoding is independent of the noise variance during training. Hence, the ***RLSSE(L,D) receiver is insensitive to white gaussian noise mismatch,*** and the MSE performance given in (29) is valid for the special case of noise mismatch. This insensitivity to noise mismatch is a significant advantage which is not offered by linear or decision feedback equalizers, as was shown in USSN 07/846,651.

Arbitrary Channel Mismatch

I can apply the same argument to the case of arbitrary channel mismatch as long as I constrain the noise 40 to be white and gaussian. Therefore, by using the arbitrary channel mismatch paradigm derived in USSN 07/846,651 and by constraining the noise 40 to be white, I find the asymptotic performance of the RLSSE(L,0) receiver is given by

MSE of RLSSE(∞,0) in Channel Mismatch and White Gaussian Noise

$$J_{RLSSE(\infty,0)} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{N_0}{|H(e^{j\omega})|^2 + N_0} + \frac{\left| \left[|H(e^{j\omega})|^2 + N_0 \right] H_T(e^{j\omega}) - \left[|H_T(e^{j\omega})|^2 + N_0 \right] H(e^{j\omega}) \right|^2}{\left[|H_T(e^{j\omega})|^2 + N_0 \right]^2 \left[|H(e^{j\omega})|^2 + N_0 \right]} \, d\omega \tag{30}$$

where $H_T(e^{j\omega})$ and $H(e^{j\omega})$ are the spectrum of the channel filter 20 during training and decoding, respectively. Now, by exploiting the arbitrary mismatch result from the related patent USSN 07/846,651, I can directly compare the performance of the RLSSE(L,0) receiver to linear and decision feedback equalizers in the presence of channel mismatch in white gaussian noise. This comparison is shown in TABLE 4 for the case of white gaussian noise mismatch and TABLE 5 for the general case of arbitary channel filter mismatch in white gaussian noise.

By comparing the theoretical performance of the RLSSE and LFE receivers, it is clear that they are identical in ideal conditions and that the RLSSE is superior for the case of white gaussian mismatch. This is also true under most practical arbitrary channel mismatch cases. The test results which follow will corroborate this important conclusion.

Test Results

My theoretical predictions and simulation results for the test channel 20 are shown in FIGURE 8 through FIGURE 12. Note that test results for other well known channels led to similar conclusions.

FIGURE 8 shows the magnitude spectrum of a typical test channel used in my tests.

FIGURE 9 shows the performance of the RLSSE receiver over the test channel shown in FIGURE 8, when no mismatch is present, i.e. ideal conditions. As a comparison, experimental data for the LFE and DFE receivers are also shown.

FIGURE 10 shows the performance of the RLSSE receiver over the test channel shown in FIGURE 8 when white gaussian noise mismatch is present. As a comparison, experimental data for the LFE and DFE receivers are also shown.

FIGURE 11 shows the performance of the RLSSE receiver over the test channel shown in FIGURE 8 when channel filter mismatch is present. As a comparison, experimental data for the LFE and DFE receivers are also shown.

FIGURE 12 shows the accuracy of my theoretical predictions for the test channel shown in FIGURE 3 when the noise variance is mismatched. Note that the same conclusions drawn from the BER curves of FIGURE 10 are made for MSE curves of FIGURE 12.

Test Conclusions and Practical Implications

There are several general observations which can be made from my described theoretical and test results. The present invention when implemented provides for robust sequence estimation in the presence of channel mismatch. The theoretical and experimental data clearly show the superiority of the present invention over standard linear and DFE receivers in mismatched conditions.

1. ***RLSSE(L,0) outperforms conventional linear equalizer.*** Theoretical and test results show the performance of the RLSSE(L,0) is comparable to that of standard linear equalizers under no mismatch (ideal) conditions. When noise mismatch is present, the RLSSE(L,0) performance far exceeded that of LFE receivers as well as the DFE receiver in some severe cases of mismatch.
2. *RLSSE(L,0) is extremely robust.* While linear and DFE receivers required tedious optimization of the training delay, no such delay was necessary or used for the linear RLSSE(L,0) receiver. Thus, its robustness with respect to channel mismatch and the algorithm parameters (L, ϵ, and γ) were extremely good.
3. ***RLSSE(L,D) is roughly equivalent to DFE over some practical channels.*** For several channels with minimal precursors, the fully-constrained RLSSE(L,L) receiver was comparable to a DFE in performance under ideal conditions and substantially better in the presence of noise mismatch over those channels. The optimization of D can also be carried out for other channels, where further improvements may also be attained by incorporating a delay in the channel estimation process and/or incorporating prefilters to mitigate precursors.
4. ***RLSSE receiver can be implemented in an extremely efficient manner.*** The same core algorithm was used for both the channel estimation and sequence estimation functions. Incorporating both channel estimator 90 and sequence estimator 70 on a single chip is possible with RLSSE, which is highly advantageous for applications such as hand-held communications, multi-media, wireless networks, and other size/weight/cost constrained systems.

System Implementation of the Preferred Embodiment

The embodiment of the present invention can be described in greater detail with reference to TABLE 1 and TABLE 3 which illustrate the recursive least squares channel estimator (RLSCE) and the recursive least squares sequence estimator (RLSSE) algorithm. It will be appreciated that this code can be implemented in software, firmware, and hardware in implementing the sequence estimator 70 and channel estimator 90. Before describing each step, note that the functions +,-, *, and / represent a means of addition, subtraction, multiplication, and division, respectively.

Channel Estimator

The algorithm of TABLE 1 is first used to estimate the channel filter 20. The preferred embodiment would use a known training sequence to estimate the channel response although alternative embodiments will be claimed as well.

STEP 0 of TABLE 1 performs an initialization of the channel estimate vector and the inverse covariance matrix.

STEP 1 of TABLE 1 provides the actual reception of the signal, $\mathbf{r}_k$ 60.

STEP 2 of TABLE 1 provides the generation of the training vector, $\mathbf{q}_k$ 330.

STEP 3 of TABLE 1 provides the generation of an intermediate variable, $\mathbf{U}_k$, which is the product of the inverse covariance matrix, $\mathbf{P}_k$, and the inverse of the exponential weighting parameter, γ. This step is introduced to reduce the overall complexity, but it is not required if $\gamma = 1$.

STEP 4 of TABLE 1 provides the generation of an intermediate variable, $\mathbf{v}_k$, which is the product of the intermediate variable $\mathbf{U}_k$, with the training vector $\mathbf{q}_k$ 330.

STEP 5 of TABLE 1 provides the calculation of the gain vector which is a function of the intermediate variable $\mathbf{v}_k$ and the training vector $\mathbf{q}_k$ 330.

STEP 6 of TABLE 1 provides the calculation of the innovation term, η_k 300.

STEP 7 of TABLE 1 provides the calculation of the inverse of the correlation matrix as a function of the intermediate variables $\mathbf{U}_k$, $\mathbf{v}_k$, and the gain vector $\mathbf{k}_k$ 310.

STEP 8 of TABLE 1 provides the calculation of the channel estimate $\tilde{\mathbf{c}}_k$ 95 as a function of the past channel estimate, $\tilde{\mathbf{c}}_{k-1}$ 320, the gain vector $\mathbf{k}_k$ 310 and the innovation term, η_k 300.

The preferred embodiment for the channel estimator would repeat STEPS 1-8 of TABLE 1 for each bit in the known training sequence. This training sequence would be inserted in the information sequence on a periodic schedule whose period is determined by the rate of change of the channel filter coefficients.

Sequence Estimator

Having found a channel estimate via the known training signal, the receiver now switches to a decoding mode, i.e., the (unknown) data is transmitted and the recursive least squares sequence estimate (RLSSE) algorithm of TABLE 3 is used to estimate that data.

STEP 0 of TABLE 3 performs an initialization of the sequence estimate vector and the inverse covariance matrix.

STEP 1 of TABLE 3 provides the actual reception of the signal, r_k 60.

STEP 2 of TABLE 3 provides the generation of the training vector, $\mathbf{q}_k$ 330. Note the only difference between this and STEP 2 of TABLE 1 illustrates the contents of the data vector, which now contains the channel estimate coefficients found from STEP 8 of TABLE 1.

STEP 3 of TABLE 3 provides the generation of an intermediate variable, $\mathbf{U}_k$, which is the product of the inverse covariance matrix, $\mathbf{P}_k$, and the inverse of the exponential weighting parameter, γ. This step is introduced to reduce the overall complexity, but it is not required if $\gamma = 1$.

STEP 4 of TABLE 3 provides the generation of an intermediate variable, $\mathbf{v}_k$, which is the product of the intermediate' variable $\mathbf{U}_k$ with the training vector $\mathbf{q}_k$ 330.

STEP 5 of TABLE 3 provides the calculation of the gain vector which is a function of the intermediate variable $\mathbf{v}_k$ and the training vector $\mathbf{q}_k$ 330.

STEP 6 of TABLE 3 provides the calculation of the innovation term.

STEP 7 of TABLE 3 provides the calculation of the inverse of the correlation matrix as a function of the intermediate variables $\mathbf{U}_k$, $\mathbf{v}_k$, and the gain vector $\mathbf{k}_k$ 310.

STEP 8 of TABLE 3 provides the calculation of the sequence estimate $\tilde{\mathbf{c}}_k$ 95 as a function of the past sequence estimate, $\tilde{\mathbf{c}}_{k-1}$ 320, the gain vector $\mathbf{k}_k$ 310 and the innovation term, η_k 300. Note that the only difference between $\tilde{\mathbf{c}}_k$ of TABLE 1 and TABLE 3 is its content which now now provides the soft sequence estimates.

STEP 9 of TABLE 3 provides picking off the oldest estimate and constraining to announce a decision.

STEP 10 of TABLE 3 provides shifting the sequence estimate and providing an initial guess for the newest received bit.

STEP 11 of TABLE 3 constrains the oldest D bits in the sequence estimate vector.

STEP 12 of TABLE 3 shifts and initializes the inverse covariance matrix $\mathbf{P}_k$.

The RLSSE receiver would process STEP 1-12 of TABLE 3 for each new received bit and would process STEP 1-8 of TABLE 1 as necessary to maintain accurate channel estimates.

It is important to note that STEPS 1- 8 of TABLE 1 and STEPS 1-8 of TABLE 3 are ***identical*** and furthermore account for the majority of all processing. Therefore, the preferred embodiment implements one core algorithm (STEPS 1-8) which can be invoked by either the channel estimator 90 or the sequence estimator 70.

In addition, since the gain vector $\mathbf{k}_k$ 310 will reach steady-state for time-invariant channels, STEPS 2-7 and STEP 12 of TABLE 3 are not necessary after an initial block of information. Thus, the preferred

embodiment would ignore these steps once the gain vector converges. Note further that this convergence has been found to occur very quickly, so that practical applications with time-varying, or slowly-time varying channels (e.g., modem, multi-media, and some wireless channels) can be processed with low complexity.

The following Tables amplify the description of my invention:

Table 1. Recursive Least Squares Channel Estimator (RLSCE) Algorithm

#	Procedure	$\pm$	$\times$	$\div$	$\geq$	I/O	Storage	Comment
0	$k = 0$, $\tilde{\mathbf{c}}_0 = \mathbf{c}_{init}$, $\mathbf{P}_0 = \varepsilon^{-1}\mathbf{I}$	-	-	-	-	-	-	Initialization
1	r_k, $k = k + 1$	0	0	0	0	1	1	Receive Signal
2	$\mathbf{q}_k$	0	0	0	0	N	N	Training Vector
3	$\mathbf{U}_k = \gamma^{-1}\mathbf{P}_{k-1}$	0	$0.5(N^2 + N)$	0	0	$0.5(N^2 + N)$	N^2	Intermediate Step
4	$\mathbf{v}_k = \mathbf{U}_k\mathbf{q}_k$	N^2	N^2	0	0	1	N	Intermediate Step
5	$\mathbf{k}_k = \dfrac{\mathbf{v}_k}{1 + \mathbf{q}_k^T\mathbf{v}_k}$	$N + 1$	N	N	0	1	N	Gain Vector
6	$\eta_k = r_k - \mathbf{q}_k^T\tilde{\mathbf{c}}_{k-1}$	$N + 1$	N	0	0	1	1	Innovation Term
7	$\mathbf{P}_k = \mathbf{U}_k - \mathbf{k}_k\mathbf{v}_k^T$	$0.5(N^2 + N)$	$0.5(N^2 + N)$	0	0	$0.5(N^2 + N)$	N^2	Correlation Matrix Inverse
8	$\tilde{\mathbf{c}}_k = \tilde{\mathbf{c}}_{k-1} + \mathbf{k}_k\eta_k$	N	N	0	0	0	N	Channel Estimate
	Total	$1.5N^2 + 3.5N + 2$	$2N^2 + 4N$	N	0	$N^2 + 2N + 4$	$2N^2 + 4N + 2$	

Notes:

1. $\tilde{\mathbf{c}}_k = \left[\tilde{a}_1 \ldots \tilde{a}_M\ \tilde{b}_0 \ldots \tilde{b}_{L-1}\right]^T$, $\mathbf{q}_k = [\,y_{k-1} \ldots y_{k-M}\ x_k \ldots x_{k-(L-1)}]^T$
2. N used in table is the order of the estimator, i.e., $N \hat{=} M + L$
3. $\mathbf{c}_{init}$ is initial estimate; ε is small positive constant; γ is exponential-weighting.
4. I/O represents memory input/output with no arithmetic operations performed.
5. Step 3 not required if $\gamma = 1.0$

Table 2. Duality Between Channel and Sequence Estimation		
	Channel Estimation	**Sequence Estimation**
Signal Model	$H(z) = \sum_{i=0}^{L-1} h_i z^{-i}$	$X(z) = \sum_{i=0}^{L-1} x_i z^{-i}$
Coefficient Vector, $\mathbf{c}_k$	$[\, h_{L-1}\ h_{L-2}\ \ldots\ h_1\ h_0]^T$	$[\, x_{k-(L-1)}\ x_{k-(L-1)-1}\ \ldots\ x_k\,]^T$
Training Vector, $\mathbf{q}_k$	$[\, x_k\ x_{k-1}\ \ldots\ x_{k-(L-1)}]^T$	$[\, \tilde{h}_{L-1}\ \tilde{h}_{L-2}\ \ldots\ \tilde{h}_1\ \tilde{h}_0]^T$
Note: An FIR channel model is shown since this is the preferred embodiment for the channel estimator function of the receiver. The duality also exists for IIR channel models, and so it can also be exploited by the RLSSE approach.		

Table 3 (Page 1 of 2). The RLSSE(L,D) Algorithm

#	Procedure	±	×	÷	≷	I/O	Storage	Comment
0	$k = 0,\ \tilde{\mathbf{c}}_0 = \mathbf{c}_{init},\ \mathbf{P}_0 = \varepsilon^{-1}\mathbf{I}$	-	-	-	-	-	-	Initialization
1	$r_k,\ k = k + 1$	0	0	0	0	1	1	Receive Signal
2	$\mathbf{q}_k$	0	0	0	0	L	L	Training Vector
3	$\mathbf{U}_k = \gamma^{-1}\mathbf{P}_{k-1}$	0	$0.5(L^2 + L)$	0	0	$0.5(L^2 + L)$	L^2	Intermediate Step
4	$\mathbf{v}_k = \mathbf{U}_k\mathbf{q}_k$	L^2	L^2	0	0	1	L	Intermediate Step
5	$\mathbf{k}_k = \dfrac{\mathbf{v}_k}{1 + \mathbf{q}_k^T\mathbf{v}_k}$	$L + 1$	L	L	0	1	L	Gain Vector
6	$\eta_k = r_k - \mathbf{q}_k^T\tilde{\mathbf{c}}_{k-1}$	$L + 1$	L	0	0	1	1	Innovation Term
7	$\mathbf{P}_k = \mathbf{U}_k - \mathbf{k}_k\mathbf{v}_k^T$	$0.5(L^2 + L)$	$0.5(L^2 + L)$	0	0	$0.5(L^2 + L)$	L^2	Correlation Matrix Inverse
8	$\tilde{\mathbf{c}}_k = \tilde{\mathbf{c}}_{k-1}^{p} + \mathbf{k}_k\eta_k$	L	L	0	0	0	L	Sequence Esti-mate
9	$\hat{x}_{k-(L-1)} = \mathrm{sgn}(\mathbf{f}\,\tilde{\mathbf{c}}_k)$	0	0	0	1	0	0	Pickoff & Con-strain
10	$\tilde{\mathbf{c}}_{k-1}^{s} = \tilde{\mathbf{c}}_{k-1}^{T}\mathbf{S} + \tilde{x}_k\mathbf{g}$	0	0	0	0	L	0	Shift & Initialize $\tilde{\mathbf{c}}_k$

Table 3 (Page 2 of 2). The RLSSE(L,D) Algorithm

#	Procedure	±	×	÷	≷	I/O	Storage	Comment
11	$\tilde{c}^{D}_{k-1} = \mathrm{sgn}_D(\tilde{c}^{s}_{k-1})$	0	0	0	D	0	0	Constrain D Symbols
12	$\mathbf{P}_k = \mathbf{S}^T\mathbf{P}_k\mathbf{S} + \varepsilon^{-1}\mathbf{gI}$	0	0	0	0	L^2	0	Shift & Initialize $\mathbf{P}_k$
	Total	$1.5L^2 + 3.5L + 2$	$2L^2 + 4L$	L	$D + 1$	$2L^2 + 3L + 4$	$2L^2 + 4L + 2$	

Notes:

1. $\tilde{\mathbf{c}}_k = [\ \tilde{x}_{k-(L-1)}\ \tilde{x}_{k-(L-1)+1}\ \ldots\ \tilde{x}_{k-1}\ \tilde{x}_k\ ]^T$, $\mathbf{q}_k = [\ \tilde{h}_{L-1}\ \tilde{h}_{L-2}\ \ldots\ \tilde{h}_1\ \tilde{h}_0]^T$
2. $\mathbf{c}_{init}$ is initial estimate; ε is small positive constant; γ is exponential-weighting; $\tilde{x}_k$ is soft estimate of current symbol.
3. Steps 1-8 are identical to the RLSCE algorithm of TABLE 1.
4. Step 3 not required if $\gamma = 1.0$
5. For time-invariant channels, Steps 2-7 and Step 12 are not necessary after the gain vector converges.

Table 4. Theoretical MSE Performance of RLSSE, LFE, and DFE In White Gaussian Noise Mismatch.

Receiver	Mean Square Error
RLSSE(∞,0)	$\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{N_0}{\lvert H(e^{j\omega})\rvert^2+N_0}d\omega$
LFE(∞,0)	$\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{N_0}{\lvert H(e^{j\omega})\rvert^2+N_0}+\frac{\Delta^2\lvert H(e^{j\omega})\rvert^2}{\left[\lvert H(e^{j\omega})\rvert^2+N_0+\Delta\right]^2\left[\lvert H(e^{j\omega})\rvert^2+N_0\right]}d\omega$
DFE(∞,∞)	$\exp\left\{\frac{1}{2\pi}\int_{-\pi}^{\pi}\ln\left[\frac{N_0}{\lvert H(e^{j\omega})\rvert^2+N_0}+\frac{\Delta^2\lvert H(e^{j\omega})\rvert^2}{\left[\lvert H(e^{j\omega})\rvert^2+N_0+\Delta\right]^2\left[\lvert H(e^{j\omega})\rvert^2+N_0\right]}\right]d\omega\right\}$

Assumptions:

1. Source is uncorrelated sequence with unit power.
2. Noise is AWGN with variance $N_0+\Delta$ during training and N_0 during decoding.
3. MSE for DFE(∞,∞) receiver assumes correct past decisions.

Table 5. Theoretical MSE Performance of RLSSE, LFE, and DFE In Channel Filter and White Gaussian Noise Mismatch.

Receiver	Mean Square Error
RLSSE(∞,0)	$\frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{N_0}{\lvert H(e^{j\omega})\rvert^2 + N_0} + \frac{\left\lvert\left[\lvert H(e^{j\omega})\rvert^2 + N_0\right]H_1(e^{j\omega}) - \left[\lvert H_1(e^{j\omega})\rvert^2 + N_0\right]H(e^{j\omega})\right\rvert^2}{\left[\lvert H_1(e^{j\omega})\rvert^2 + N_0\right]^2\left[\lvert H(e^{j\omega})\rvert^2 + N_0\right]}\, d\omega$
LFE(∞,0)	$\frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{N_0}{\lvert H(e^{j\omega})\rvert^2 + N_0} + \frac{\left\lvert\left[\lvert H(e^{j\omega})\rvert^2 + N_0\right]H_1(e^{j\omega}) - \left[\lvert H_1(e^{j\omega})\rvert^2 + N_0 + \Delta\right]H(e^{j\omega})\right\rvert^2}{\left[\lvert H_1(e^{j\omega})\rvert^2 + N_0 + \Delta\right]^2\left[\lvert H(e^{j\omega})\rvert^2 + N_0\right]}\, d\omega$
DFE(∞,∞)	$\exp\left\{\frac{1}{2\pi}\int_{-\pi}^{\pi} \ln\left[\frac{N_0}{\lvert H(e^{j\omega})\rvert^2 + N_0} + \frac{\left\lvert\left[\lvert H(e^{j\omega})\rvert^2 + N_0\right]H_1(e^{j\omega}) - \left[\lvert H_1(e^{j\omega})\rvert^2 + N_0 + \Delta\right]H(e^{j\omega})\right\rvert^2}{\left[\lvert H_1(e^{j\omega})\rvert^2 + N_0 + \Delta\right]^2\left[\lvert H(e^{j\omega})\rvert^2 + N_0\right]}\right] d\omega\right\}$

Assumptions:

1. Source is uncorrelated sequence with unit power.
2. Noise is AWGN with variance $N_0 + \Delta$ during training and N_0 during decoding. Channel filter spectrum is $H_1(e^{j\omega})$ and $H(e^{j\omega})$ during training and decoding, respectively.
3. MSE for DFE(∞,∞) receiver assumes correct past decisions.

While I have described my preferred embodiments of my invention, it will be understood that those skilled in the art, both now and in the future, may make make various improvements and enhancements which fall within the scope of the claims which follow. These claims should be construed to maintain the proper protection for the invention first disclosed.

Claims

1. A receiving system for use with an adaptive receiver for providing an output signal $\hat{x}_{k-(L-1)}$ (80) in response to an input signal r_k (60) received from a discrete-time equivalent channel which has a linear channel filter (20) and additive white gaussian noise (40), said receiver system comprising:
   a sequence estimator and a means for providing a channel estimate;
   said sequence estimator (70) having as inputs a received signal r_k and a channel filter estimate (95) whose L coefficients are determined by said means for providing a channel estimate, and as output the hard decision $\hat{x}_{k-(L-1)}$ (80) which is an estimate of the transmitted information sequence x_k (10) according to the recursive relation:

$\mathbf{U}_k = \gamma^{-1}\mathbf{P}_{k-1}$
$\mathbf{v}_k = \mathbf{U}_k\mathbf{q}_k$

$$\mathbf{k}_k = \frac{\mathbf{v}_k}{1 + \mathbf{q}_k^T \mathbf{v}_k}$$

$$\eta_k = r_k - \mathbf{q}_k^T \tilde{\mathbf{c}}_{k-1}$$

$$\mathbf{P}_k = \mathbf{U}_k - \mathbf{k}_k\mathbf{v}_k^T$$

$$\tilde{\mathbf{c}}_k = \tilde{\mathbf{c}}_{k-1}^D + \mathbf{k}_k\eta_k$$

$$\hat{x}_{k-(L-1)} = \mathrm{sgn}(\mathbf{f}\,\tilde{\mathbf{c}}_k)$$

$$\tilde{\mathbf{c}}_{k-1}^S = \tilde{\mathbf{c}}_{k-1}^T\mathbf{S} + \tilde{x}_k\mathbf{g}$$

$$\tilde{\mathbf{c}}_{k-1}^D = \mathrm{sgn}_D(\tilde{\mathbf{c}}_{k-1}^S)$$

$\mathbf{P}_k = \mathbf{S}^T\mathbf{P}_k\mathbf{S} + \epsilon^{-1}\mathbf{g}\mathbf{I}$

where γ is the desired exponential weighting parameter for the sequence estimator, $\mathbf{P}_{k-1}$ is an L by L inverse correlation matrix initialized to $\epsilon^{-1}\mathbf{I}$, $\mathbf{I}$ is the L by L identity matrix, ϵ is the desired initialization parameter for the sequence estimator, sgn(z) denotes a binary threshold device which returns a one (1) if its parameter z is greater than or equal to zero and minus one (-1) if z is less than zero, $\tilde{\mathbf{c}}_k$ is the sort-estimate vector of the received sequence initialized to an arbitrary vector, $\mathrm{sgn}_D(z)$ denotes the D-threshold device which applies the sgn-function to the left-most D bits in the vector $\mathbf{z}$, $\mathbf{f}$ is the L-length pick-off vector given by

$\mathbf{f}$ = [1 0 ... 0]

$\mathbf{g}$ is the L-length initialization vector given by

$\mathbf{g}$ = [0 ...0 1]

$\mathbf{S}$ is the L by L shift matrix given by

$$\mathbf{S} = \begin{bmatrix} 0 & 0 & & 0 & 0 \\ 1 & 0 & & 0 & 0 \\ 0 & 1 & \cdots & \vdots & \vdots \\ \vdots & \vdots & & 0 & 0 \\ 0 & 0 & & 1 & 0 \end{bmatrix}$$

2. A receiver system according to claim 1, further comprising: as a means for providing a channel estimate, a channel estimator (90), having as inputs the received signal r_k (60) and a training vector

$$\mathbf{q}_k^T$$

(330) whose coefficients are the past L known training bits x_k (10), and as output the channel filter estimate $\tilde{\mathbf{c}}_k$ (95) according to the relation

$\mathbf{U}_k = \gamma^{-1}\mathbf{P}_{k-1}$
$\mathbf{v}_k = \mathbf{U}_k\mathbf{q}_k$

$$\mathbf{k}_k = \frac{\mathbf{v}_k}{1 + \mathbf{q}_k^T\mathbf{v}_k}$$

$$\eta_k = r_k - \mathbf{q}_k^T\tilde{\mathbf{c}}_{k-1}$$

$$\mathbf{P}_k = \mathbf{U}_k - \mathbf{k}_k\mathbf{v}_k^T$$

$$\tilde{\mathbf{c}}_k = \tilde{\mathbf{c}}_{k-1} + \mathbf{k}_k\eta_k$$

where γ is the desired exponential weighting parameter for the channel estimator, $\mathbf{P}_{k-1}$ is $\mathbf{L}$ by $\mathbf{L}$ the inverse correlation matrix initialized to $\epsilon^{-1}\mathbf{I}$, ϵ is the desired initialization parameter for the channel estimator, $\tilde{\mathbf{c}}_k$ is the current estimate of the channel fitter coefficients which is initialized at start-up to a vector which corresponds to the nominal channel filter expected during implementation.

3. A receiver system according to Claim 1 wherein the means for providing a channel estimate and sequence estimate which exploit the same software/hardware to perform the co functions defined by the recursive relation

$\mathbf{U}_k = \gamma^{-1}\mathbf{P}_{k-1}$
$\mathbf{v}_k = \mathbf{U}_k\mathbf{q}_k$

$$\mathbf{k}_k = \frac{\mathbf{v}_k}{1 + \mathbf{q}_k^T \mathbf{v}_k}$$

$$\eta_k = r_k - \mathbf{q}_k^T \tilde{\mathbf{c}}_{k-1}$$

$$\mathbf{P}_k = \mathbf{U}_k - \mathbf{k}_k \mathbf{v}_k^T$$

$$\tilde{\mathbf{c}}_k = \tilde{\mathbf{c}}_{k-1} + \mathbf{k}_k \eta_k$$

4. Receiver system as claimed in Claim 1, 2 or 3, wherein the means for providing a channel estimate is obtained off-line via experimentation and/or channel models.

5. Receiver system as claimed in Claim 1, 2, 3 or 4, wherein sequence estimating or channel estimating functions are optimized for a particular architecture and/or compiler and/or channel while providing the relations expressed by Claim 1, or
wherein sequence estimating or channel estimating functions are optimized for a particular architecture and/or compiler and/or channel while providing the relations expressed by Claim 2.

6. Receiver system as claimed in any one of the Claims 1 to 5, wherein the channel estimator or sequence estimate are implemented in whole or in part by a combination of hardware, firmware and/or software.

7. Receiver system as claimed in Claim 1 or one of the Claims 2 to 6, wherein the receiver system is reduced in complexity due to known channel conditions or assumptions, selected from a group consisting of
a knowledge/assumption of white noise; and/or a
knowledge/assumption of time-invariant channel filter; and/or a
knowledge/assumption of stationary noise statistics; and/or a
knowledge/assumption of channel or noise models.

8. Receiver system as claimed in Claim 7, wherein the receiver system may incorporate stationary channels, where the gain vector $\mathbf{k}_k$ of Claim 1 is known to converge, and hence only the following relations of Claim 1 are necessary after the gain vector $\mathbf{k}_k$ converges to a steady-state value:

$$\eta_k = r_k - \mathbf{q}_k^T \tilde{\mathbf{c}}_{k-1}$$

$$\tilde{\mathbf{c}}_k = \tilde{\mathbf{c}}_{k-1}^D + \mathbf{k}_k \eta_k$$

$$\hat{x}_{k-(L-1)} = \mathrm{sgn}(\mathbf{f}\,\tilde{\mathbf{c}}_k)$$

$$\tilde{\mathbf{c}}_{k-1}^S = \tilde{\mathbf{c}}_{k-1}^T \mathbf{S} + \tilde{x}_k \mathbf{g}$$

$$\tilde{\mathbf{c}}_{k-1}^D = \mathrm{sgn}_D(\tilde{\mathbf{c}}_{k-1}^S)$$

9. Receiver system according to Claim 1 or one of the Claims 2 to 8, wherein the sequence estimator is modified to provide sequence estimates of non-binary data.

10. Receiver system as claimed in Claim 2 or one of the related Claims 4 to 9, wherein the sequence and channel estimators provide sequence estimates over complex channels.

11. Receiver system as claimed in Claim 2 or one of the related Claims 4 to 10, wherein the sequence estimator or channel estimator use variants of recursive least squares algorithms, said recursive least squares algorithms preferably including: fast RLS algorithms, lattice implementations, and square-root or Cholesky factorization methods.

12. Receiver system as claimed in Claim 1 or one of the related Claims 2 to 11, wherein the channel estimator is implemented in batch form by the relation

$$\tilde{\mathbf{c}}_k = [\mathbf{Q}_k^T \mathbf{W}_k \mathbf{Q}_k]^{-1} \mathbf{Q}_k^T \mathbf{W}_k \mathbf{r}_k$$

where the observable vector $\mathbf{r}_k$ (230) represents the L-length vector with the past L received signals $\mathbf{r}_k$ - (60), the training matrix $\mathbf{Q}$ is the $\mathbf{L}$ by L matrix whose rows are the past L training vectors $\mathbf{q}_k^T$ whose contents are given by the past L known training bits, $\mathbf{W}_k$ is an arbitrary weighting matrix, and $\mathbf{r}_k$ is the observable vector which is updated with each new bit by the relation

$$\mathbf{r}_k^T = \mathbf{r}_{k-1}^T \mathbf{S} + r_k \mathbf{g}$$

13. Receiver system as claimed in Claim 1 or one of the claims 2 to 12, wherein the sequence estimator is implemented in batch form by the relation

$$\tilde{\mathbf{c}}_k = [\mathbf{Q}_k^T \mathbf{W}_k \mathbf{Q}_k]^{-1} \mathbf{Q}_k^T \mathbf{W}_k \mathbf{r}_k$$

$$\tilde{x}_{k-(L-1)} = \mathbf{f}\, \tilde{\mathbf{c}}_k$$

where the observable vector $\mathbf{r}_k$ (230) represents the L-length vector with the past L received signals $\mathbf{r}_k$ - (60), the training matrix $\mathbf{Q}$ is the L by L matrix whose rows are the past L training vectors $\mathbf{q}_k^T$ whose contents are given by the channel estimator, $\mathbf{W}_k$ is an arbitrary weighting matrix, and $\mathbf{r}_k$ is the observable vector which is updated with each new bit by the relation

$$\mathbf{r}_k^T = \mathbf{r}_{k-1}^T \mathbf{S} + r_k \mathbf{g}$$

14. Receiver system as claimed in Claim 12 or 13, wherein the channel and sequence estimator exploit the same software/hardware to perform the core function given by the relation

$$\tilde{\mathbf{c}}_k = [\mathbf{Q}_k^T \mathbf{W}_k \mathbf{Q}_k]^{-1} \mathbf{Q}_k^T \mathbf{W}_k \mathbf{r}_k$$

15. Receiver system as claimed in Claim 1 or one of the related preceding Claims 2 to 14, wherein a RLSSE algorithm is used for sequence estimation of transmitted data which has been coded by a

scheme such as, but not limited to linear filtering, convolutional codes, and trellis-coded modulation techniques.

16. Receiver system as claimed in Claim 1 or one of the related preceding Claims 2 to 15, wherein the operators **S**, **f**, and **g** are implemented in software/hardware as memory shift operations.

17. Receiver system as claimed in Claim 1 or one of the related preceding Claims 2 to 16, wherein the means for providing a channel estimate is a channel estimator (90) which uses past decisions instead of a known training signal in order to update the channel estimate.

18. Receiver system for evaluation of a received signal, comprising:
an input port for a received signal having characteristics that need evaluation including two signal sequences, one of which is an unknown information sequence and another of which is an unknown channel filter effect,
the receiver system having an output providing an estimate of the unknown information sequence, and a common functional evaluation unit for evaluating the received signal performing the functions of a sequence estimator and a means for providing a channel estimate.

19. Receiver system according to Claim 18, wherein said means for providing a channel estimate is provided by an external prior evaluation, and said channel estimator is a functional unit which provides an estimate of a channel filter effect on said received signal.

20. Receiver system according to Claim 18, wherein said means for providing a channel estimate is provided by an internal channel estimator portion of said receiver system, and said receiver system also includes a sequence estimator portion which is combined with said channel estimator portion to provide said common functional unit which provides an estimate of a channel filter effect on said received signal and an estimate of an information sequence utilizing steps for evaluation of said information sequence in common with steps for evaluation of the channel filter effect estimate of received signal.

21. Receiver system according to Claim 20, wherein said steps for evaluation of said information sequence in common are defined by the recursive relation

$\mathbf{U}_k = \gamma^{-1}\mathbf{P}_{k-1}$
$\mathbf{v}_k = \mathbf{U}_k\mathbf{q}_k$

$$\mathbf{k}_k = \frac{\mathbf{v}_k}{1 + \mathbf{q}_k^T \mathbf{v}_k}$$

$$\eta_k = r_k - \mathbf{q}_k^T \tilde{\mathbf{c}}_{k-1}$$

$$\mathbf{P}_k = \mathbf{U}_k - \mathbf{k}_k \mathbf{v}_k^T$$

$$\tilde{\mathbf{c}}_k = \tilde{\mathbf{c}}_{k-1} + \mathbf{k}_k \eta_k.$$

or by the relation

$$\tilde{\mathbf{c}}_k = [\mathbf{Q}_k^T \mathbf{W}_k \mathbf{Q}_k]^{-1} \mathbf{Q}_k^T \mathbf{W}_k \mathbf{r}_k.$$

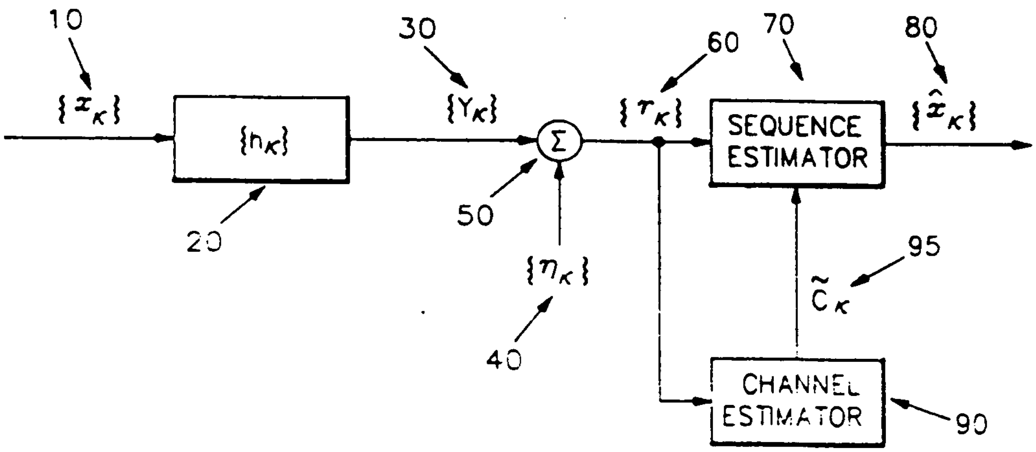
10
$\{x_K\}$
$\{h_K\}$
20
30
$\{Y_K\}$
50
$\{\eta_K\}$
40
60
$\{r_K\}$
70
SEQUENCE ESTIMATOR
80
$\{\hat{x}_K\}$
95
$\tilde{C}_K$
CHANNEL ESTIMATOR
90

FIG.1

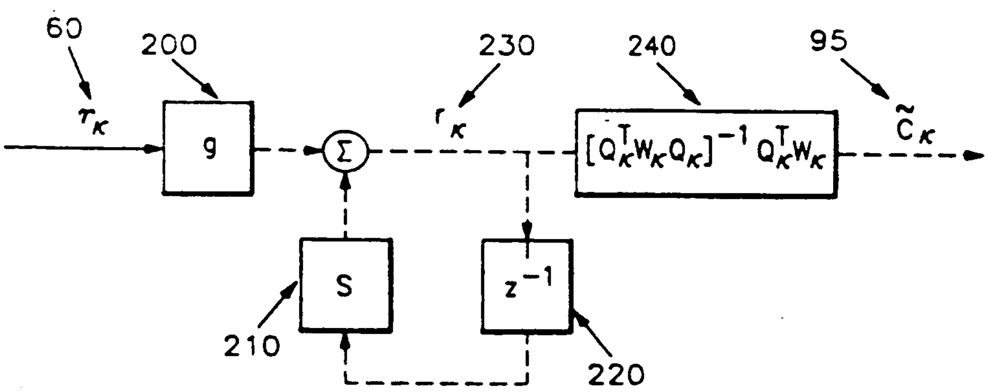
60
r_K
200
g
210
S
230
r_K
220
z^{-1}
240
$[Q_K^T W_K Q_K]^{-1} Q_K^T W_K$
95
$\tilde{C}_K$

FIG.2

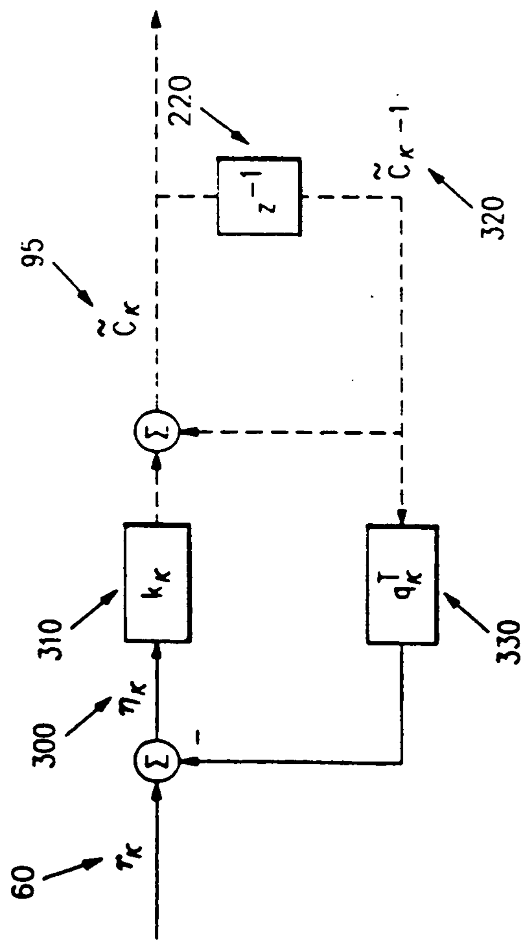
60
r_K
300
n_K
310
k_K
330
q_K^T
95
$\tilde{C}_K$
220
z^{-1}
320
$\tilde{C}_K - 1$

FIG.3

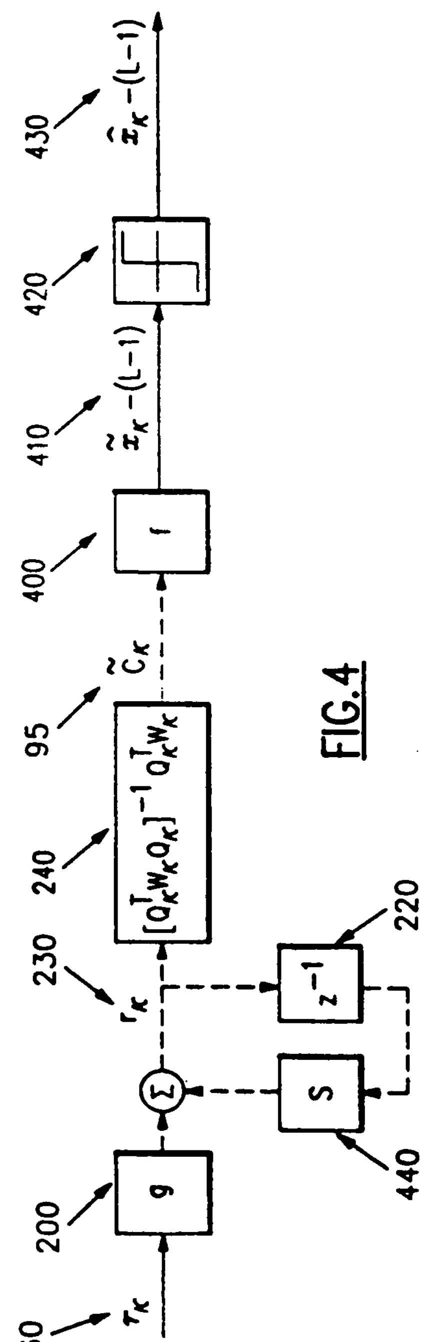
60
r_K
200
g
440
S
220
z^{-1}
230
r_K
240
$[q_K^T W_K q_K]^{-1} q_K^T W_K$
95
$\tilde{C}_K$
400
f
410
$\tilde{x}_K-(L-1)$
420
430
$\hat{x}_K-(L-1)$

FIG.4

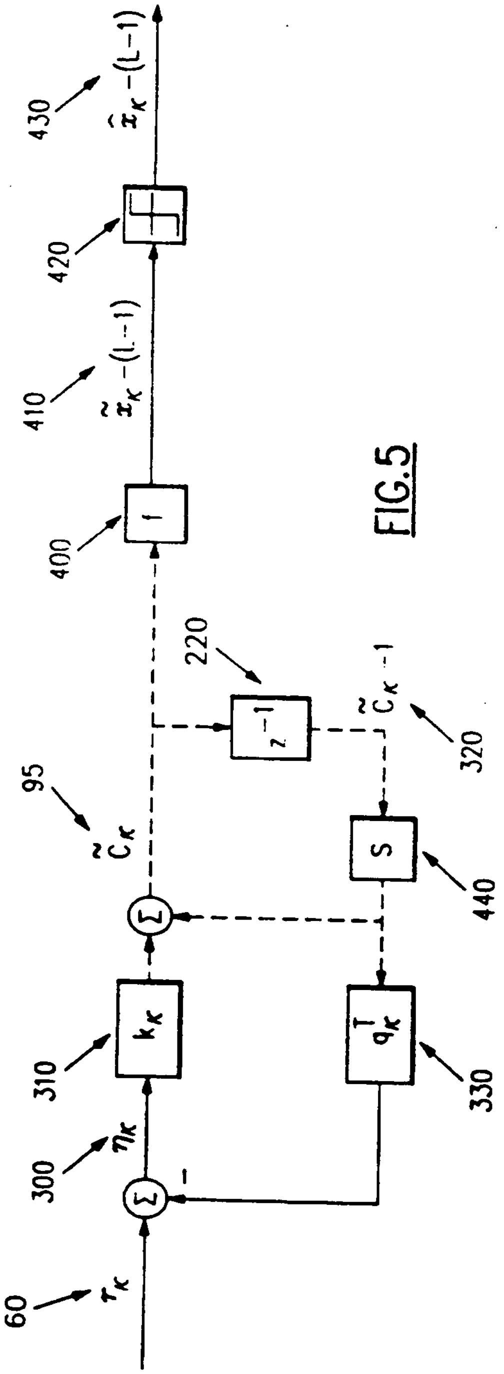
60
300
310
95
220
320
440
330
400
410
420
430

FIG.5

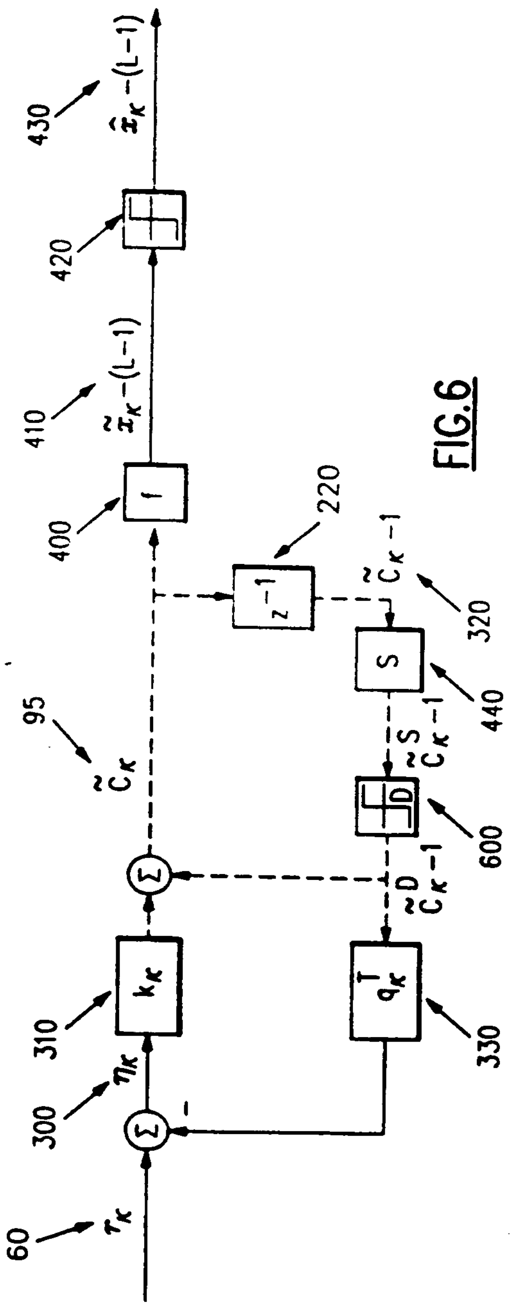
60
300
310
95
220
320
440
600
330
400
410
420
430

FIG.6

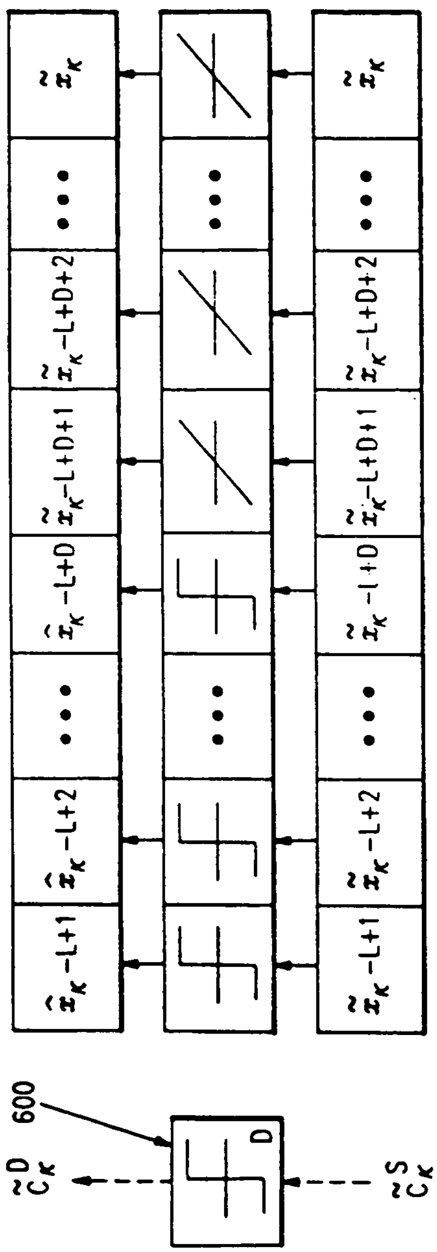
600
D
$\tilde{C}_K^D$
$\tilde{C}_K^S$
$\hat{x}_K$-L+1
$\hat{x}_K$-L+2
$\hat{x}_K$-L+D
$\tilde{x}_K$-L+D+1
$\tilde{x}_K$-L+D+2
$\tilde{x}_K$
$\tilde{x}_K$-L+1
$\tilde{x}_K$-L+2
$\tilde{x}_K$-L+D
$\tilde{x}_K$-L+D+1
$\tilde{x}_K$-L+D+2
$\tilde{x}_K$

FIG.7

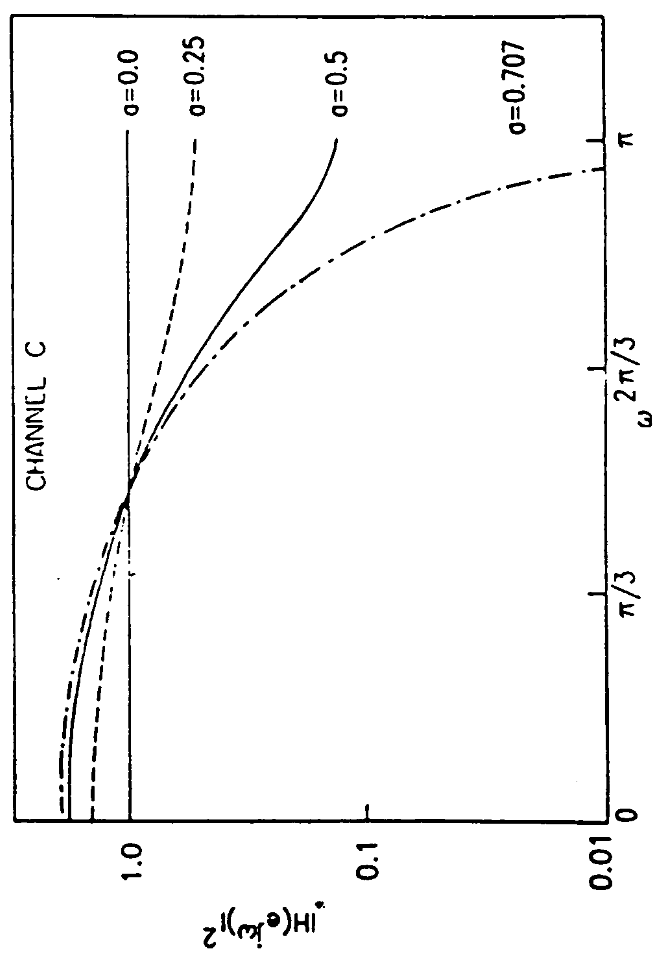
CHANNEL C
a=0.0
a=0.25
a=0.5
a=0.707
1.0
0.1
0.01
0
π/3
2π/3
π
ω
$|H(e^{j\omega})|^2$

FIG.8

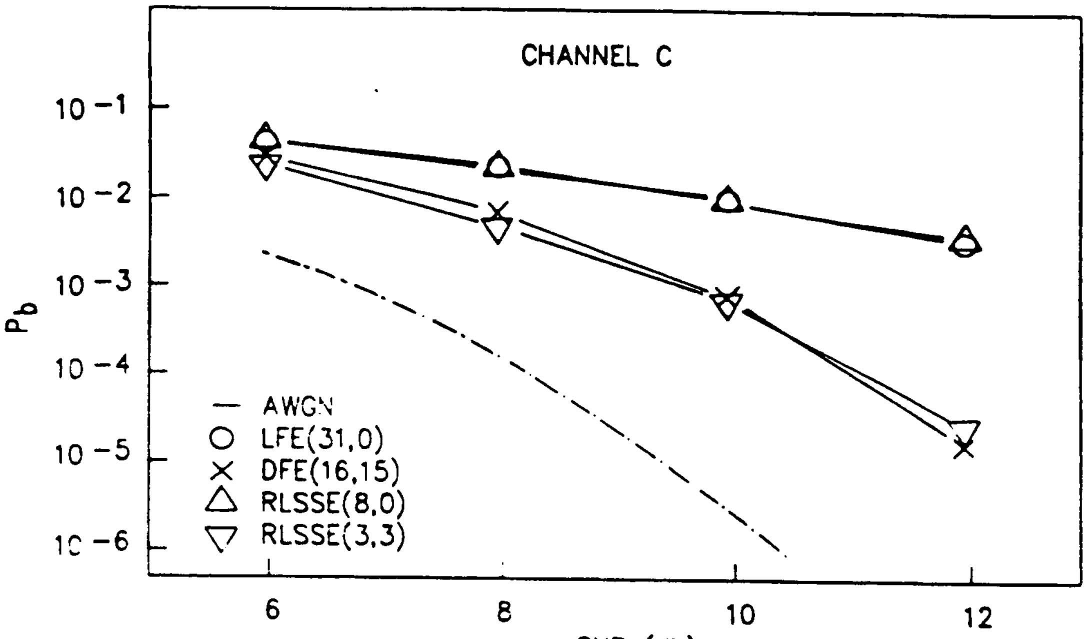
CHANNEL C
10 −1
10 −2
10 −3
10 −4
10 −5
10 −6
P_b
AWGN
LFE(31,0)
DFE(16,15)
RLSSE(8,0)
RLSSE(3,3)
6
8
10
12
SNR (db)

FIG.9

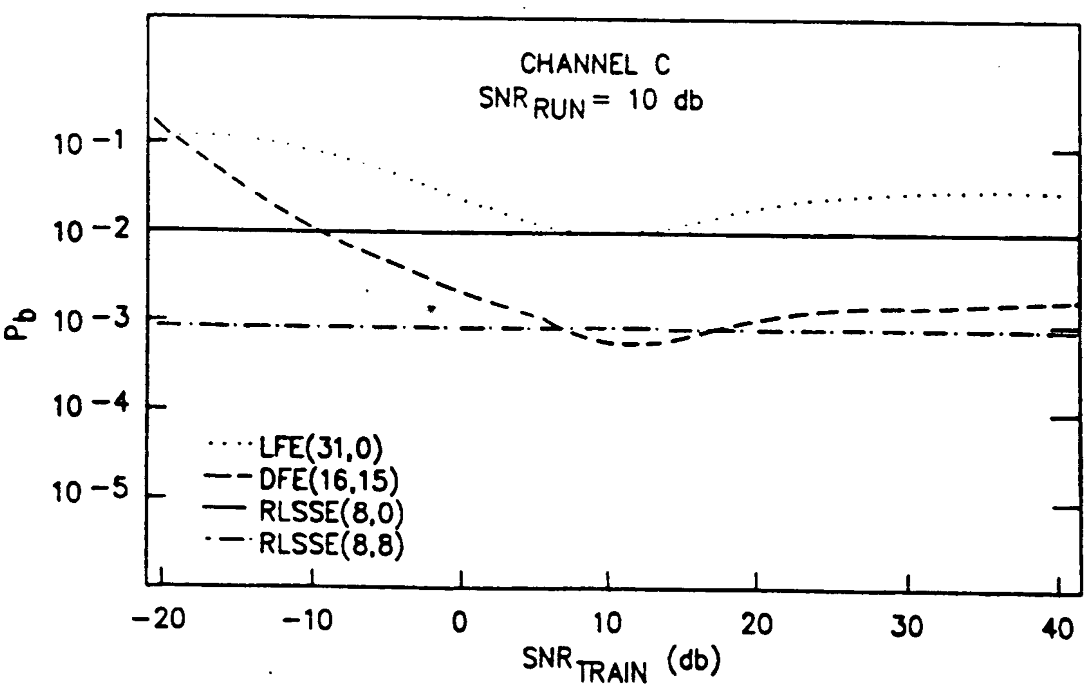
CHANNEL C
SNR_{RUN} = 10 db
10 −1
10 −2
10 −3
10 −4
10 −5
P_b
LFE(31,0)
DFE(16,15)
RLSSE(8,0)
RLSSE(8,8)
−20
−10
0
10
20
30
40
SNR_{TRAIN} (db)

FIG.10

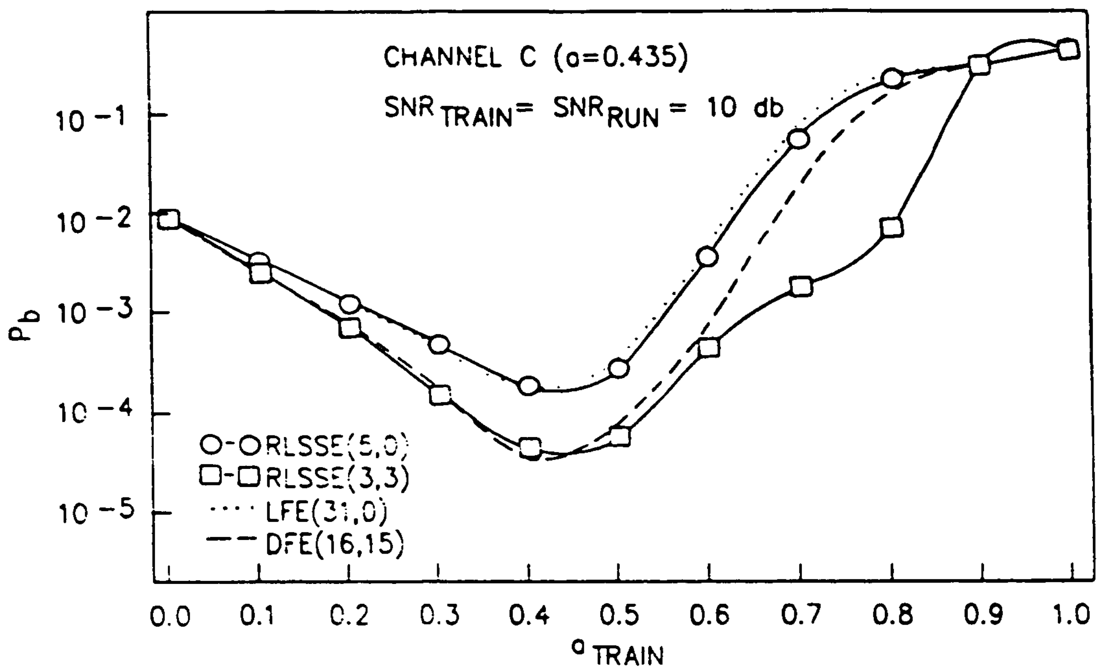
CHANNEL C (a=0.435)
SNR TRAIN= SNR RUN = 10 db
10 −1
10 −2
10 −3
10 −4
10 −5
Pb
O-O RLSSE(5,0)
□-□ RLSSE(3,3)
···· LFE(31,0)
— — DFE(16,15)
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
a TRAIN

FIG.11

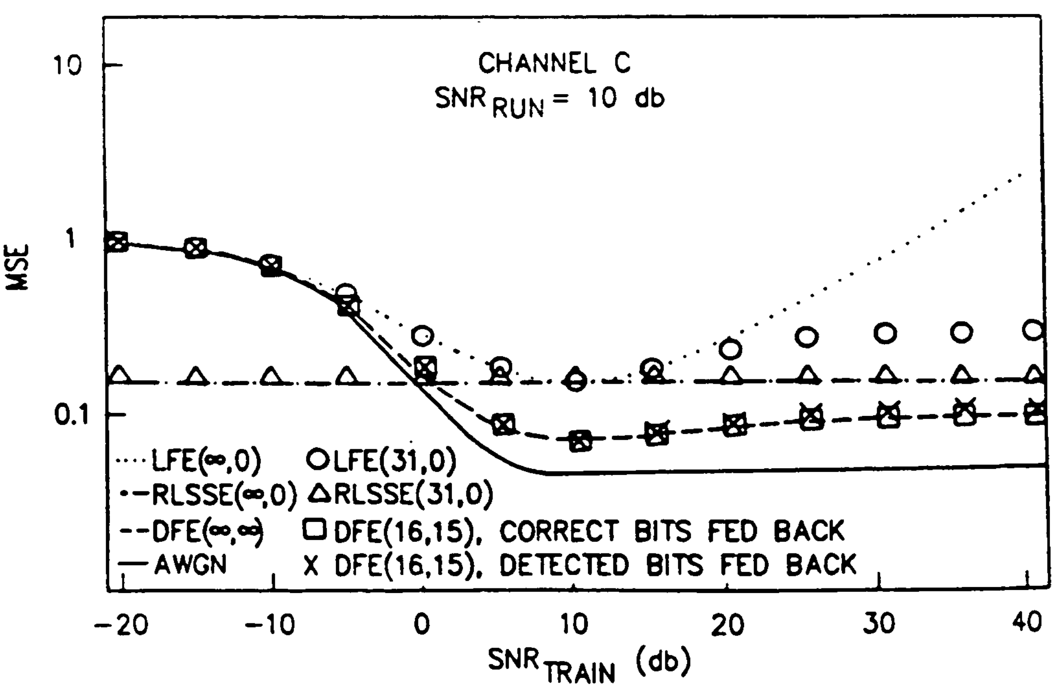
CHANNEL C
SNR RUN = 10 db
10
1
0.1
MSE
····LFE(∞,0) O LFE(31,0)
•–RLSSE(∞,0) △RLSSE(31,0)
--DFE(∞,∞) □ DFE(16,15), CORRECT BITS FED BACK
—AWGN X DFE(16,15), DETECTED BITS FED BACK
−20 −10 0 10 20 30 40
SNR TRAIN (db)

FIG.12

European Patent Office

PARTIAL EUROPEAN SEARCH REPORT

which under Rule 45 of the European Patent Convention shall be considered, for the purposes of subsequent proceedings, as the European search report

Application Number

EP 93 10 4157

Page 1

DOCUMENTS CONSIDERED TO BE RELEVANT

Category	Citation of document with indication, where appropriate, of relevant passages	Relevant to claim	CLASSIFICATION OF THE APPLICATION (Int. Cl. 5)
A,D	CSELT RAPPORTI TECNICI no. 3, April 1974, pages 3 - 22 LUVISON AND PIRANI 'A state - variable approach to the equalization of multilevel digital signals' * page 5, left column, paragraph 3 - page 6, left column, paragraph 1 * * page 11, left column, paragraph 2 - page 12, left column, paragraph 3; figures 11,12 * ---	1,6,8, 12,14, 16,18-20	H04L25/03
A	IEEE International Conference on Communications, 11-14th June 1989, Boston, US; IEEE, New York, US, 1989; pages 193 - 197, McLaughlin et al.: "A performance study of 3 adaptive equalisers in the mobile communication environment" * abstract * * page 193, right column, paragraph 4 - page 194, left column, paragraph 2 * ---	1,6,8, 12,14, 16,18-20	
	-/--		TECHNICAL FIELDS SEARCHED (Int. Cl. 5) H04L

INCOMPLETE SEARCH

The Search Division considers that the present European patent application does not comply with the provisions of the European Patent Convention to such an extent that it is not possible to carry out a meaningful search into the state of the art on the basis of some of the claims

Claims searched completely :

Claims searched incompletely :

Claims not searched :

Reason for the limitation of the search:

see sheet C

Place of search	Date of completion of the search	Examiner
THE HAGUE	01 JULY 1993	SCRIVEN P.

CATEGORY OF CITED DOCUMENTS

X : particularly relevant if taken alone
Y : particularly relevant if combined with another document of the same category
A : technological background
O : non-written disclosure
P : intermediate document

T : theory or principle underlying the invention
E : earlier patent document, but published on, or after the filing date
D : document cited in the application
L : document cited for other reasons

& : member of the same patent family, corresponding document

European Patent Office

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Application Number

EP 93 10 4157

DOCUMENTS CONSIDERED TO BE RELEVANT

Category	Citation of document with indication, where appropriate, of relevant passages	Relevant to claim	CLASSIFICATION OF THE APPLICATION (Int. Cl. 5)
A,D	IEEE TRANSACTIONS ON COMMUNICATION TECHNOLOGY vol. 19, no. 6, December 1971, NEW YORK US pages 1137 - 1141 LAWRENCE AND KAUFMAN 'The Kalman filter for the equalization of a digital communications channel' * abstract * * page 1138, left column, paragraph 5 - right column, paragraph 2 * * page 1139, left column, paragraph 5 - right column, paragraph 1 * ---	1,6,8, 12,14, 16,18-20	
A	OPTICAL ENGINEERING vol. 31, no. 6, June 1992, pages 1224 - 1228 GHOSH AND WEBER 'Maximum - likelihood blind equalization' * abstract * ---	1,6,8, 12,14,16	TECHNICAL FIELDS SEARCHED (Int. Cl. 5)
A,D	IEEE TRANSACTIONS ON COMMUNICATIONS vol. 35, no. 10, October 1987, NEW YORK US pages 1037 - 1040 PRASAD AND PATHAK 'An approach to recursive equalization via white sequence estimtion techniques' * page 1037, left column, paragraph 1 * ---	1,6,8, 12,14,16	
A,D	IEEE TRANSACTIONS ON COMMUNICATIONS vol. 22, no. 9, September 1974, NEW YORK US pages 1205 - 1215 BENEDETTO AND BIGLIERI 'On linear receivers for digital transmission systems' * page 1206, right column, paragraph 4 - page 1207, right column, paragraph 1 * ---	1,6,8, 12,14,16	
A,D	ELECTRONICS LETTERS vol. 16, no. 14, 3 July 1980, ENAGE GB pages 547 - 549 DALLE MESE AND CORSINI 'Adaptive Kalman filter equaliser' * abstract * * page 548, right column, paragraph 3; figures 1,2 * --- -/--	1,6,8, 12,14,16	

European Patent Office

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Application Number

EP 93 10 4157

DOCUMENTS CONSIDERED TO BE RELEVANT			
Category	Citation of document with indication, where appropriate, of relevant passages	Relevant to claim	CLASSIFICATION OF THE APPLICATION (Int. Cl. 5)
A,D	IEEE TRANSACTIONS ON AEROSPACE AND ELECTRONIC SYSTEMS vol. 15, no. 5, September 1979, NEW YORK US pages 635 - 648 LUVISON AND PIRANI 'Design and performance of an adaptive Kalman receiver for synchronous data transmission' * page 636, left column, paragraph 4 - page 637, left column, paragraph 1 * -----	1,6,8, 12,14,16	
			TECHNICAL FIELDS SEARCHED (Int. Cl. 5)

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INCOMPLETE SEARCH

Claims searched completely : 1-3,6,8,12,14,16,18-21
Claims not searched : 4,5,7,9-11,13,15,17

Reason : Claims 4,5,7,9-11,13,15,17 contain subject matter, which is not supported by the description.